FUZZY LOGIC CONTROL Vs. CONVENTIONAL PID CONTROL OF AN INVERTED PENDULUM ROBOT

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ABSTRACT

This paper addresses some of the potential benefits of using fuzzy logic controllers to control an inverted pendulum system. The stages of the development of a fuzzy logic controller using a four input Takagi-Sugeno fuzzy model were presented. The main idea of this paper is to implement and optimize fuzzy logic control algorithms in order to balance the inverted pendulum and at the same time reducing the computational time of the controller. In this work, the inverted pendulum system was modeled and constructed using Simulink and the performance of the proposed fuzzy logic controller is compared to the more commonly used PID controller through simulations using Matlab. Simulation results show that the Fuzzy Logic Controllers are far more superior compared to PID controllers in terms of overshoot, settling time and response to parameter changes.

I INTRODUCTION

It has been traditional for roboticists to mimic the human body. The human body is so perfect in many ways that it seems like a long way before a robot will ever get close to exactly representing a human body.

One of the less thought about issues in robotics is the issue of balance, which can be appropriately represented by the balancing act of an inverted pendulum. This explains the fact that although many investigations have been carried out on the inverted pendulum problem [1]-[10], researchers are still constantly experimenting and building it as the inverted pendulum is a stepping stone to greater balancing control systems such as balancing robots.

Therefore, in order to control the balancing act of the inverted pendulum, a control system is needed. As known, fuzzy logic control systems model the human decision making process based on rules and have become popular elements as they are inexpensive to implement, able to solve complicated nonlinear control problems and display robust behavior compared to the more commonly used conventional PID control systems [1, 2, 11].

In general, there are numerous and various control problems such as balancing control systems which involve phenomena that are not amenable to simple mathematical modeling. As known, conventional control system which relies on the mathematical model of the underlying system has been successfully implemented to various simple and non-linear control systems. However, it has not been widely used with complicated, non-linear and time varying systems [3, 4, 5, 11]. On the other hand, fuzzy logic is a powerful and excellent analytical method with numerous applications in embedded control and information processing. Fuzzy provides a straightforward and easy path to describe or illustrate specific outcomes or conclusions from vague, ambiguous or imprecise information [1].

Review on existing conventional and fuzzy logic techniques has highlight the significance and importance of control systems. Researchers have proven that fuzzy logic control systems are able to overcome nonlinear control problems which may not be solved easily using conventional methods and the delicate process in designing a fuzzy logic controller that is able to mimic the human experience and knowledge in controlling a system. Therefore, it will be interesting to show that fuzzy logic controllers are able to control many of these problems without having a clear understanding of the underlying phenomena and are more favorable and superior compared to conventional PID controllers.

This paper presents the systematic design of a fuzzy logic controller using the Takagi-Sugeno model for a car-pendulum mechanical system, well-known as the inverted pendulum problem. Here, the inverted pendulum and control system is first modeled before putting them into simulations using Matlab where the control system is further tuned to increase its performance. The control system is then further optimized in order to reduce the computational time of the system by reducing the number of rule bases. This is followed by the implementation and comparison of the PID and fuzzy controllers through simulations.

II. INVERTED PENDULUM MODEL

Fig. 1 shows the block diagram of an inverted pendulum system with a feedback fuzzy control block. The output of the plant $(\theta, \dot{\theta}, x, \dot{x})$ is fed into the controller to produce the subsequent force to balance the pendulum to its upright position and at the same time maintaining the cart initial position.

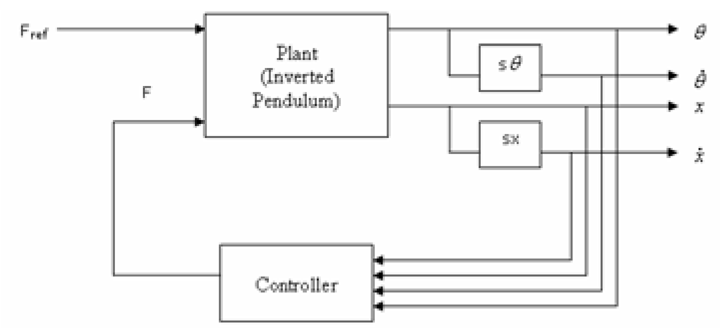


Fig. 1. Block diagram of inverted pendulum system with feedback fuzzy logic controller.

The inverted pendulum system consists of a moving cart and a pivoted bar that is free to oscillate in the x-y plane. However, the cart is constrained to move only in the x-plane as shown in Fig. 2. In Fig. 2, m_C is the mass of the cart, m_P is the mass of the pendulum, μ is the coefficient of friction, g is the acceleration of gravity and I is the moment of inertia of the pendulum about the pivot.

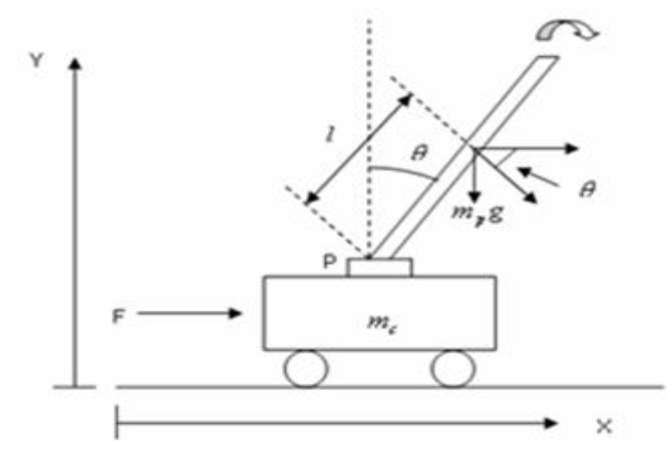


Fig 2: Inverted Pendulum System

From Fig. 2 and the pendulum's free body diagram, the state equations in terms of the control force, F can be expressed as,

$$\ddot{x} = \frac{\frac{4}{3}F - \frac{4}{3}m_p l\dot{\theta}^2 \sin\theta - m_p g \sin\theta\cos\theta}{\frac{4}{3}(m_c + m_p) - m_p l \cos^2\theta} \qquad (1)$$

$$\ddot{\theta} = \frac{(m_c + m_p) g \sin\theta - F\cos\theta + m_p l\dot{\theta}^2 \sin\theta\cos\theta}{\frac{4}{3}(m_c + m_p) l - m_p l \cos^2\theta} \qquad (2)$$

where length, $L = 2l$ and $I = 4/3\, m_p l^2$. θ is the falling angle,

$\dot{\theta}$ is the angular velocity, $\ddot{\theta}$ is the angular acceleration of the pendulum. $\ddot{x}$ is the acceleration of the cart.

Since the control force, F in terms of the motor voltage, V can be expressed as [6],

$$F = \frac{K_m K_g}{Rr} V - \frac{K_m^2 K_g^2}{Rr^2}\dot{x}$$

where $\dot{x}$ is the velocity of the cart, K_m is the motor torque constant, K_g is the gearbox ratio, R is the motor armature resistance and r is the motor pinion radius, the state equations for the inverted pendulum in terms of the motor voltage, V has been derived as,

$$\ddot{x} = \frac{\frac{4}{3}\left(\frac{K_m K_g}{Rr} V - \frac{K_m^2 K_g^2}{Rr^2}\dot{x}\right) - m_p g\theta}{\frac{4}{3}(m_c + m_p) - m_p}$$

$$\ddot{\theta} = \frac{(m_c + m_p) g\theta - \frac{K_m K_g}{Rr} V + \frac{K_m^2 K_g^2}{Rr^2} \dot{x}}{\frac{4}{3}(m_c + m_p) l - m_p l} \qquad (5)$$

The model of the inverted pendulum is then created using Simulink. The input to the plant is the disturbance force imparted to the cart. The current angle and angular velocity is fed back to the system to calculate the angular acceleration and acceleration of the cart and pendulum respectively. The angular acceleration and the acceleration are then integrated to obtain the outputs of angle, angular acceleration, position and velocity.

As a whole, the balancing algorithm (controller) measures two outputs from the plant (Inverted Pendulum) and calculates the torque forces, F needed for balance. Fig. 3 shows how the forces, F are determined from the angle, angular velocity, position and velocity measured from their respective sensors.

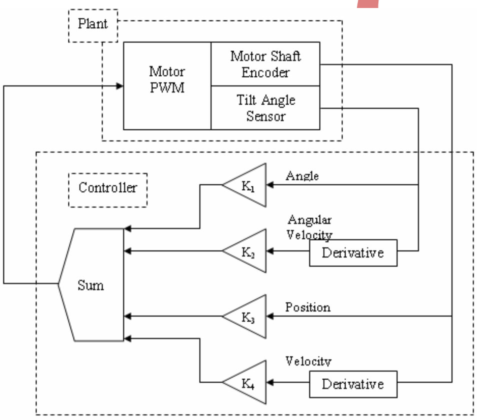


Fig. 3. Plant and controller block diagram.

Fig. 3 shows that the motor shaft encoder measures the position of the cart's wheel while the angle sensor measures the tilt angle of the pendulum. The angular velocity and the velocity of the cart are then derived respectively from the measured tilt angle and the cart's position. The four triangles K_1, K_2, K_3 and K_4 are the "knobs" that apply gain to the four feedback signals. They are summed together and fed back to the system as the PWM motor voltage to drive the cart. This can be expressed as,

$$F = (\text{Angle}, \theta \times K_1) + (\text{AngularVelocity}, \dot{\theta} \times K_2) + (\text{Position}, x \times K_3) + (\text{Velocity}, \dot{x} \times K_4) \qquad (6)$$

The controller input gains, K_1, K_2, K_3 and K_4 are determined using the Linear-Quadratic Regulator (LQR) method described by Friedland [11]. This method finds the optimal K based on the state feedback law and the state-space equation derived earlier. It is found that the input gains, K1, K2, K3 and K4 respectively are approximately 40, 10, 3 and 4.

III. FUZZY LOGIC SYSTEM

Fuzzy controllers are very simple. They consist of an input stage, a processing stage and an output stage. The input or fuzzification stage maps sensor or other inputs to the appropriate membership functions and truth values. The processing or the rule evaluation stage invokes each appropriate rule and generates a result for each, then combines the results of the rules.

The output or the defuzzification stage then converts the combined result back into a specific control output value using the Centroid Method.

Fig. 4 shows the fuzzy inference process as discussed below

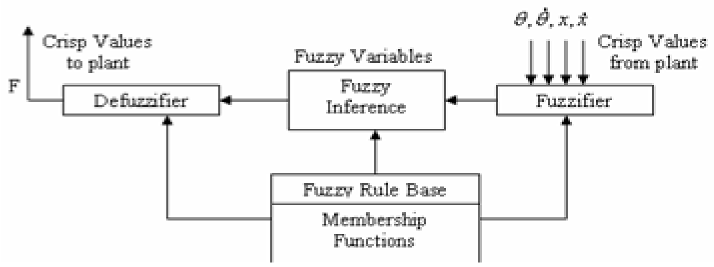
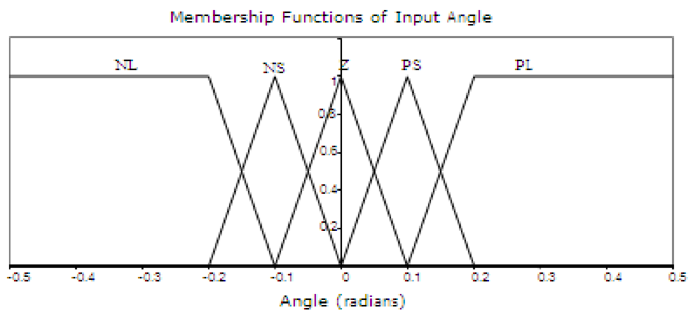
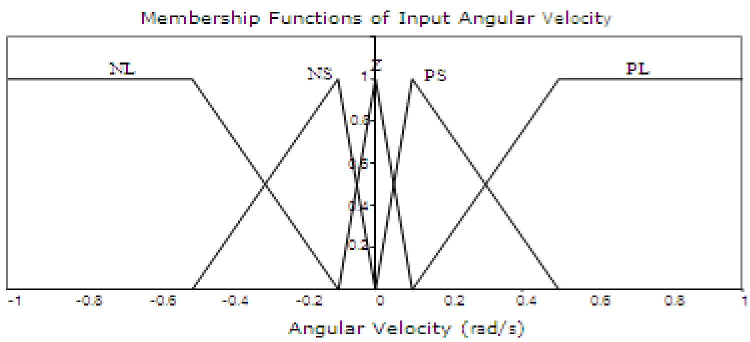


Fig. 4. Fuzzy Control Process.

while Fig. 5 shows the shape and range of the membership functions for the input angle, angular velocity, position and

The input variables NL, NS, Z, PS and PL for the inputs angle and angular velocity represent the membership functions of Negative Large, Negative Small, Zero, Positive Small and Positive Large respectively. While the input variables N, Z and P for the inputs position and velocity represent the membership functions Negative, Zero and Positive respectively.





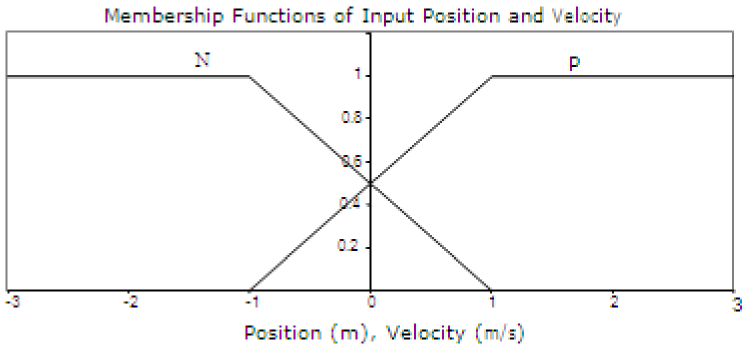


Fig. 5. Membership functions of inputs.

For this particular problem, an output window that consists of 13 fuzzy singletons is used. Each fuzzy singleton is a linear function that defines the output of the system as given in (7).

Output force, $F = K_1\theta + K_2\dot{\theta} + K_3 x + K_4 \dot{x} + K_5$

where K_5 is a constant.

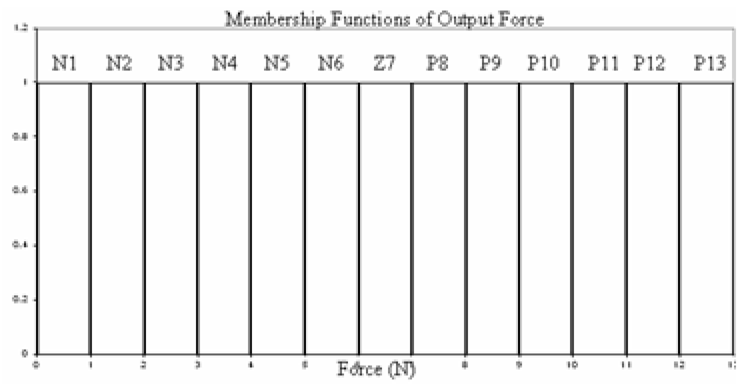


Fig. 6 shows the membership functions of the output force

while Table 1 represents the function of each membership function defined in Fig. 6. The variables N1 to N6 represent a negative force while variables P8 to P13 represent a positive force and Z7 represents zero force Values of gains K_1 to K_5 in Table 1 are obtained through tuning of the input and output membership functions based on the assumptions as follows:

- Output force is non linear. At larger angle, output force is larger.
- Influence of input θ towards output > Influence of $\dot{\theta}$ > Influence of x and $\dot{x}$.
- Constant K5 is required for the fine tuning of the output.
- A lower overshoot response and shorter settling time is desired.

Table 2 on the other hand defines the relationship between the input variables and the output variable which is the required force to balance the pendulum.

The rule base is constructed based on the assumptions as follows. Considering in terms of the inputs angle or angular velocity,

- The larger the –ve input, the larger the –ve force.
- The larger the +ve input, the larger the +ve force.
- At zero input, the force depends on the magnitude of the other three inputs.

TABLE 1

OUTPUT FORCE & INPUT GAINS

Variables	K_1	K_2	K_3	K_4	K_5
N1	41.4	10.03	3.16	4.29	0.34
N2	40.4	10.05	3.16	4.29	0.21
N3	41.4	10.03	3.16	4.29	0.34
N4	40.4	10.05	3.16	4.29	0.21
N5	38.6	10.18	3.16	4.29	-0.05
N6	37.6	10.15	3.16	4.29	-0.18
Z7	37.6	10.15	3.16	4.29	0.00
P8	37.6	10.15	3.16	4.29	0.18
P9	38.6	10.18	3.16	4.29	0.05
P10	40.4	10.05	3.16	4.29	-0.21
P11	41.4	10.03	3.16	4.29	-0.34
P12	40.4	10.05	3.16	4.29	-0.21
P13	41.4	10.03	3.16	4.29	-0.34

TABLE 2

RULE BASE

$x:\dot{x}$ / $\theta:\dot{\theta}$	N:N	N:Z	N:P	Z:N	Z:Z	Z:P	P:N	P:Z	P:P
NL:NL	N5	N4	N3	N4	N3	N2	N3	N2	N1
NL:NS	N6	N5	N4	N5	N4	N3	N4	N3	N2
NL:Z	Z7	N6	N5	N6	N5	N4	N5	N4	N3
NL:PS	P8	Z7	N6	Z7	N6	N5	N6	N5	N4
NL:PL	P9	P8	Z7	P8	Z7	N6	Z7	N6	N5
NS:NL	N6	N5	N4	N5	N4	N3	N4	N3	N2
NS:NS	Z7	N6	N5	N6	N5	N4	N5	N4	N3
NS:Z	P8	Z7	N6	Z7	N6	N5	N6	N5	N4
NS:PS	P9	P8	Z7	P8	Z7	N6	Z7	N6	N5
NS:PL	P10	P9	P8	P9	P8	Z7	P8	Z7	N6
Z:NL	Z7	N6	N5	N6	N5	N4	N5	N4	N3
Z:NS	P8	Z7	N6	Z7	N6	N5	N6	N5	N4
Z:Z	P9	P8	Z7	P8	Z7	N6	Z7	N6	N5
Z:PS	P10	P9	P8	P9	P8	Z7	P8	Z7	N6
Z:PL	P11	P10	P9	P10	P9	P8	P9	P8	Z7
PS:NL	P8	Z7	N6	Z7	N6	N5	N6	N5	N4
PS:NS	P9	P8	Z7	P8	Z7	N6	Z7	N6	N5
PS:Z	P10	P9	P8	P9	P8	Z7	P8	Z7	N6
PS:PS	P11	P10	P9	P10	P9	P8	P9	P8	P7
PS:PL	P12	P11	P10	P11	P10	P9	P10	P9	P8
PL:NL	P9	P8	Z7	P8	Z7	N6	Z7	N6	N5
PL:NS	P10	P9	P8	P9	P8	Z7	P8	Z7	N6
PL:Z	P11	P10	P9	P10	P9	P8	P9	P8	Z7
PL:PS	P12	P11	P10	P11	P10	P9	P10	P9	P8
PL:PL	P13	P12	P11	P12	P11	P10	P11	P10	P9

Considering in terms of the inputs position or velocity,

- The larger the +ve input, the larger the –ve force.
- The larger the –ve input, the larger the +ve force.
- At zero input, the force depends on the magnitude if the other three inputs.

Therefore, by assigning the weighting values of 1 to 5 respectively to NL, NS, Z, PS, and PL for inputs angle and angular velocity and weighting values of 1 to 3 respectively to N, Z and P for inputs position and velocity, the rule base is determined using the formula,

$$F = \theta + (\dot{\theta} - 1) + (-x + 3) + (-\dot{x} + 3) \qquad (8)$$

where F denotes the values 1 to 13 as represented in the output membership functions in Fig. 6 while $\theta, \dot{\theta}$, x,

$\dot{x}$ denotes the weighting values of the respective inputs corresponding to the membership functions.

Equation (8) is derived in such a way that:

- The addition of θ_i and $\dot{\theta}$ as well as the subtraction of in the calculation of F_i correspond to the direction of the inputs with respect to the direction of F as explained above.
- The subtraction of the value 1 from $\dot{\theta}$ shows that the influence of θ_i is greater than the influence of θ_i towards the required force.
- The addition of the value 3 to x_i and $\dot{x}_I$ is necessary based on how the weighting values are assigned.

The weighting values area signed based (7) in such a way that:

- The higher the weighting values of inputs angle and angular velocity, the larger the force in the positive direction.
- The higher the weighting values of inputs position and velocity, the larger the force in the negative direction.

For example, given the conditions as follow:

- The falling angle of the pendulum is large in the positive direction.
- The pendulum is falling slowly (small angular velocity) in the negative direction.
- The cart is moving in the negative direction at the negative position.

$\therefore F_i = 5 + (2 - 1) + (-1 + 3) + (-1 + 3) = 10$ (Refer to Table 2)

IV. SIMULATION

The inverted pendulum and fuzzy controller systems modeled are then implemented and simulated in the Matlab environment using Simulink and the Fuzzy Logic Toolbox. The fuzzy logic controller modeled was first implemented using the graphical user interface in the fuzzy logic toolbox. As the fuzzy logic toolbox is designed to work flawlessly with Simulink, the system is then embedded directly into simulation.

Fig. 7 shows the fuzzy logic control system of the inverted pendulum system. In this system, a step response is used as the reference position. The system works base on the concept that at a smaller step reference, the force required to move the pendulum to that reference position is smaller and vice versa. This force is known as the disturbance force or the reference force as mentioned earlier which will be imparted to the inverted pendulum system.

Due to this disturbance force, the pendulum will bethe feedback path will then try to balance the pendulum evaluating the current state of the angle and the angular velocity of the pendulum and the current position and the velocity of the cart to produce the appropriate force and direction to balance the pendulum back to its upright position.

The fuzzy control system here was first implemented, tuned and optimized using the fuzzy logic toolbox before being implemented into the fuzzy logic controller. The inverted pendulum plant on the other hand consists of several masks layer that defines the whole inverted pendulum system. Due to this, the physical specifications that define the pole length, pole mass, cart mass and the acceleration due to gravity, g can be modified to test the system and controller's performance at different conditions.

In order to reduce the system's computational time, the 225 rule based designed previously are optimized and reduced to 16 rule base. The optimization process is simplified as the Sugeno-type inference is used in this fuzzy logic controller.

The optimization process involves the reduction of the number of membership functions of all the inputs to two membership functions and also the reduction of the 225 rules to 16 only as discussed earlier.

Table 3 shows the newly optimized rule base

TABLE 3

OPTIMIZED RULE BASE

$\theta : \dot{\theta}$ \ $x : \dot{x}$	N:N	N:P	P:N	P:P
N:N	N1	N2	N3	N4
N:P	N5	N6	N7	N8
P:N	P9	P10	P11	P12
P:P	P13	P14	P15	P16

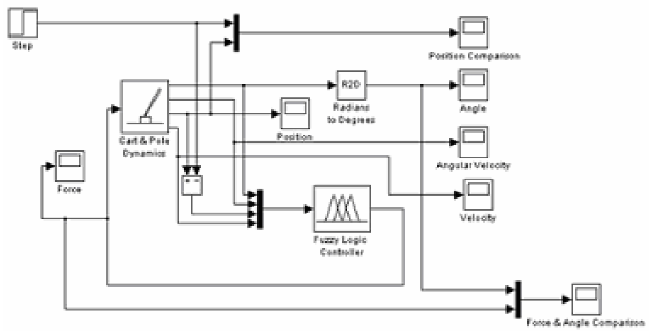


Fig. 7. Simulink model of inverted pendulum problem with fuzzy logic control.

V RESULTS AND DISCUSSION

This section discusses the simulation development results and compares the fuzzy logic controller performance at different physical conditions. The optimized simulation results are also compared with the previous results to show that there is only a very slight degradation of performance in the controller when the rule base of the system is reduced and tuned to reduce the computational time of the controller. Besides that, the sensors are also tested to show the relationship between the falling angle of the pendulum and the speed of the motors.

5.1 Simulation Results

The Simulink model in Fig. 8 is simulated with relative tolerance of 0.001s. Fig. 8 and 9 present the simulation for the inputs angle and position and the output control force with pole mass of 0.1kg, cart mass of 1.0kg, pole length of 1.0m, input step reference of 1.0 and acceleration due to gravity of 9.8ms^{-2}. Due to the step response, the actual simulation of the system only starts at t=1s. In other words, the simulation of the system only starts when the disturbance force is applied to the system. Fig. 9 shows that the cart is able to reach its desired position in about 6s while balancing the pendulum. The desired position here represents the reference position of the system which is 1 step in this case.

Fig. 8 on the other hand shows that with a step response of 1, a disturbance force of about 3N in the opposite direction is applied to the system. Therefore, in order for the cart to reach its desired or reference position in the positive direction and at the same time balancing the pendulum at the shortest time, a negative disturbance force is applied to the system. When a negative force is applied, the pendulum will be displaced to the positive direction due to the inertia of the pendulum.

Due to this, the fuzzy logic controller will apply the appropriate force in the positive direction to balance the pendulum. This explains the force and the angle response at the start of the simulation and the very small negative displacement of the cart's position as shown in Fig. 8. The response also shows that the falling angle of the pendulum requires 6.5s to return to its upright position of zero angle.

Fig. 10 shows the simulation of the optimized system. The only difference between the previous and the optimized system is that the optimized system takes an extra time of 0.5s to balance the pole to its upright position.

5.2 Fuzzy Control vs. Conventional PID Control

Fig. 11 and Fig. 12 show two sets of results comparing the application of fuzzy control and conventional control (PID controller) techniques to the inverted pendulum problem simulation. For the same system parameters here, the PID controller proportional gain, Kp, derivative gain, Kd and integral gain, Ki are found to be 9, 14, and 0.06 respectively. The first two graph show that the fuzzy logic controller gives a smaller overshoot and shorter settling time. In the second set, the mass of the cart is changed without modifying the controllers. Fig. 14 shows that the conventional controller totally failed to balance the pendulum as it was designed for the nominal value of cart mass. On the other hand, the fuzzy logic controller exhibited small performance degradation due to this parameter change as shown in Fig. 13. This proves that fuzzy logic is not based on the mathematical model of the inverted pendulum and more robust to mass variations.

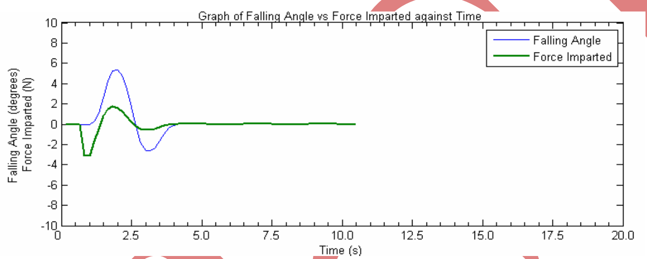


Fig. 8. Falling angle versus force imparted response.

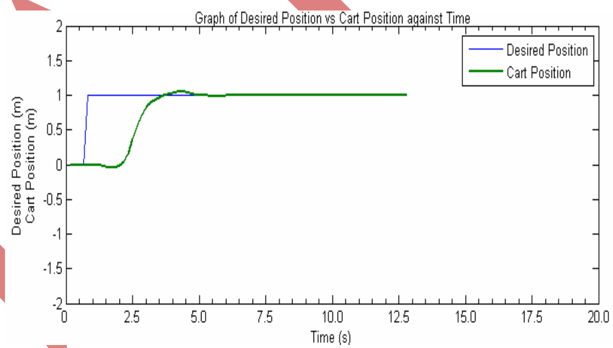


Fig. 9. Desired position versus cart position response.

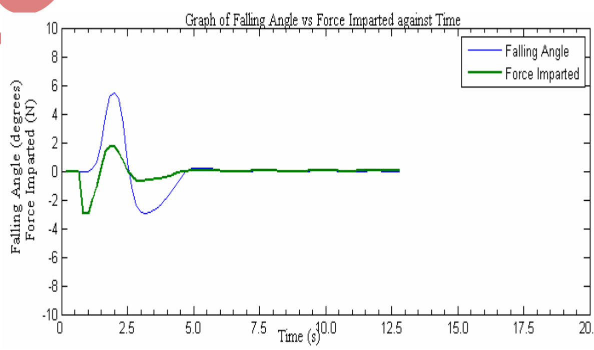


Fig. 10. Optimized falling angle versus force imparted response.

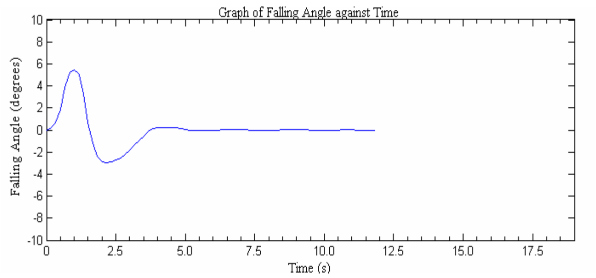


Fig. 11. Falling angle response for fuzzy control.

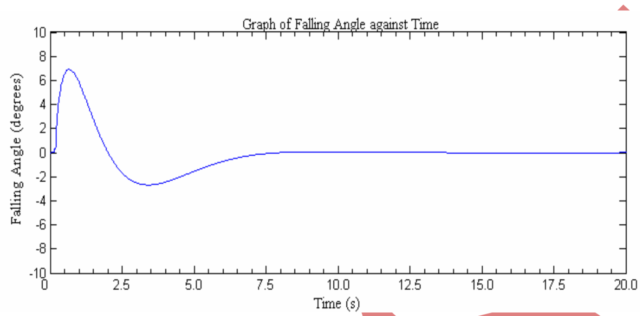


Fig. 12. Falling angle response for conventional PID control.

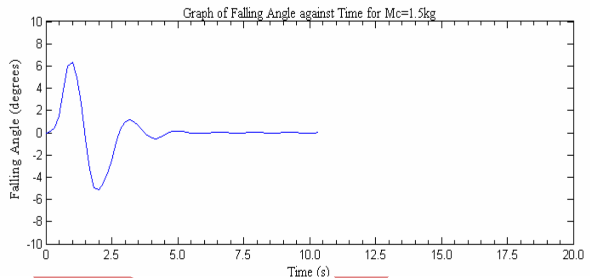


Fig. 13. Falling angle response for fuzzy control (Mass changed)

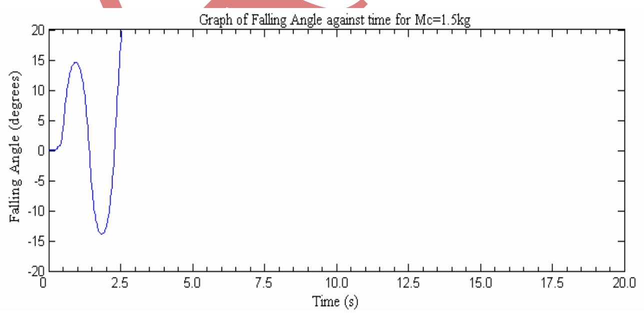


Fig. 14. Falling angle response for conventional PID control (Mass changed).

VI. CONCLUSIONS

In this paper, an optimized fuzzy logic controller has been implemented in the Matlab environment, using the Fuzzy Logic Toolbox and Simulink. It has been used to control an inverted pendulum system. The study has identified some of the potential benefit of using fuzzy logic controllers. In comparison with the modern control theory, fuzzy logic is simpler to implement as it eliminates the complicated mathematical modeling process and uses a set of control rules instead. The achieved results showed that proposed fuzzy logic controller is more robust to parameter variations when compared to the PID controller.

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