# EFFECT OF VOLUME FRACTION AND FIBER ORIENTATION ON MODAL CHARACTERISTICS OF COMPOSITE BEAM

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## ABSTRACT

*Now a day's laminated composite beams are commonly preferred for various engineering applications on priority basis due to more suitability and advantages over metallic beams. These beams are often subjected to different working environments and loads during their operation. Hence the Dynamic analyses of composite components are quite essential to avoid the failure of the component, which can be forecasted and analyzed by the determination of natural frequency and mode shape. The use of the composite structures according to the need of the applications decides the end conditions and geometry of the composite beam which plays an important role during deciding the mechanical properties like stiffness and rigidity of the composite material. Also it is true that the mechanical properties depend upon the orientation of fiber layer, boundary condition and geometrical shape of the beam which in turns reflects on the natural frequency. The mechanical property of a composite material can be expressed as a function of fiber and matrix volume fraction. In this paper the effect of variation of fiber volume fraction on mechanical properties as a function of natural frequency is studied for different boundary condition of the beam for different stacking sequences or layer orientations of the layers using finite element analysis software.*

***Keywords: Boundary Condition, Dynamic Analysis, E-Glass, Fiber Orientations, Laminated Composite, Natural Frequency***



## I INTRODUCTION

The strength of a composite structure depends on its constituent fiber and matrix material. Also for same matrix and fiber combination, variation of volume fraction, presence of void and variation of geometry of the structure plays important role for the variation of mechanical properties. The orientation of fiber layer, thickness and boundary condition also controls the elastic modulus and strength of the composite material. The use of the composite specimen according to its necessity laid different boundary condition and geometry for it.

A number of researchers had carried out numerous numbers of methods for dynamic analysis of pate like composite beams. Lee and Yhim [3] analyzed single and two-span continuous composite plate structures subjected to multi-moving loads using 7-DOF finite element model for computational analysis and third order shear deformation theory to validate. Koo [4] studied the effects of layer wise in-plane displacements on fundamental frequencies and specific damping capacity for composite laminated plates using FEA method and experimental method. Majid et.al. [5] had developed frequency response function and modal analysis for composite plate like wing and tested it as cantilever to get the dynamic properties and its dependency on ply orientation and thickness. Matsunaga [6] determined the natural frequencies and buckling stresses of cross-ply laminated composite plates under the influence of shear deformation, thickness change and rotary inertia using the method of power series expansion of displacement components and Hamilton's principle for above. Davallo et al. [7] studied the mechanical behavior of uni-directional glass-polyester composites in flexure and tensile testing. The effect of laminate thickness on the mechanical properties is studied using simple energy model. Mohammed et.al [8] had explained the effect of fiber orientations on the flexural natural frequencies by finite element (FEA) and experimental approach. Yung et al. [9] presented transient dynamic finite element analysis of laminated composite under the influence of transverse load by the use of Newmark scheme and Newton-Raphson method. Liou et al. [10] made an investigation over the transient response of an E-glass epoxy laminated composite plate impacted by a steel circular cylinder by three-dimensional hybrid stress finite element program to determine the transverse deflection at centre. Lee et al. [11] investigated the dynamic behavior of multiply-folded composite laminates using higher order plate theory and the third order finite element program. The effects of folding angles and ply orientations on the transient responses for various loading and boundary conditions are studied. Morozov [12] has made a theoretical and experimental characterization of elastic properties of the textile composites. Ratnaparkhi and Sarnobat [13] made the modal analysis to obtain the Natural frequencies in free-free boundary condition and validated the results obtained from the FEA using ANSYS in their work. Khalili et al. [14] used Fourier series to investigate the dynamic response of laminated composite plate subjected to static and dynamic loading The result is validated by comparing with the result obtained from the FEM code NISA II. Attaran et al. [15] made a study over the effects of aspect ratio, sweep angle, and stacking sequence of laminated composites on aero dynamic properties like flutter speed using 2D finite element analysis in conjunction with Doublet lattice Method. Jweeg et al.[16] made experimental and theoretical study on modulus of elasticity of composite material due to the reinforcement of different types of fiber like short, long woven, powder, and particulate shapes. Long et al. [17] presented a general formulation for free and transient vibration analyses of composite laminated beams for any boundary condition. To confirm the validity of the formulation, the result is compared with the result obtained from the analytical, experimental and FEA. Parida and Dash [18,19,20] developed generalized frequency equation taking bending and shearing in to consideration and studied the effect of fiber orientation, boundary condition and aspect ratio on modal characteristics of a plate like beam using finite element analysis software.

From the above study it is evident that the orientation of fiber layer and thickness controls the elastic modulus and strength of the composite material. In this paper the effect of volume fraction and fiber orientation on natural frequency for different boundary conditions of the beam is to be studied. In section 2 mathematical modeling is proposed for free vibration analysis of Timoshenko composite beam. The problem for the analysis is defined and validation of FEA analysis is done in this section also. In section 3 effect of different fiber orientation and fiber volume fraction on natural frequency for different boundary condition are investigated in a detail.

## II MATHEMATICAL FORMULATION

The modal analysis of a beam in general refers to the determination of natural frequency and mode shape of the beam. In general beams the bending phenomenon is pre-dominant. The natural frequency for a beam subjected to bending action only is given by

$$(\omega_{zi}^{B})^2 = \frac{EI_{zz}}{2\pi\rho}\frac{\alpha_{Bi}^4}{L^4} \quad (1)$$

However for a beam undergoing shearing action, the free natural frequency for a beam subjected to shearing is given by

$$(\omega_{zi}^{s})^2 = \frac{S_{yy}}{\rho}\frac{\alpha_{si}^2}{L^4} \quad (2)$$

Where, $S_{yy} = \frac{5}{6} B \int_{T/2}^{T/2} (q_{ss} dy)$, $EI_{zz} = \frac{1}{a_{xx}}\frac{B^3}{12}$ in $N.m^2$

The super script s stands for shearing and B stands for bending

In this work, the beam is considered as Timoshenko beam. In Timoshenko beam the shear deformation is considered for beam analysis. The first order shear deformation theory (FSDT) is used for beam analysis. For analysis, it is assumed that the cross sections of beam subjected to bending remain plane but not perpendicular to the axis of application of bending load. Here the effect of bending and shearing is considered. The natural frequencies of the orthotropic beams (as shown in figure-1) subjected to bending and shearing deformation simultaneously by shear beam theory is expressed as

$$(\omega_{zi})^2 = \left[\frac{1}{(\omega_{zi}^{B})^2} + \frac{1}{(\omega_{zi}^{s})^2}\right]^{-1} \quad (3)$$

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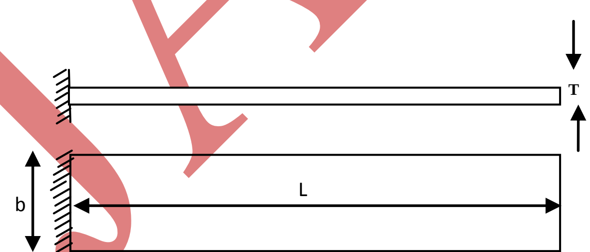


**Figure-1. Figure showing dimension of Cantilever beam**

The structure of the composite material is depicted in figure-2. The composite beam of thickness T is constituted by n number of plies each of equal thickness.

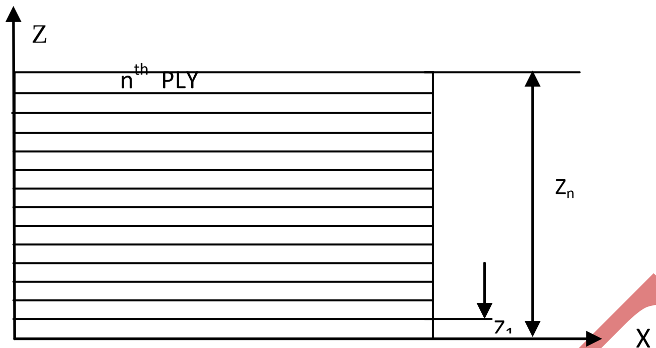


**Figure-2. Multidirectional laminate with co-ordinate notation of individual plies**

The mechanical properties of the woven fabric composite beams as a function of volume fraction are given by.

$$\left.\begin{aligned}
U_i &= U_f V_f + U_m V_m \\
V_f &= \frac{\text{volume of fibers}}{\text{total volume of composite}} \\
V_{\tilde{v}} &= 1 - \frac{\left(\frac{W_f}{\rho_f}\right) + \frac{W_c - W_f}{\rho_m}}{\frac{W_c}{\rho_c}} \\
\rho_c &= \rho_f v_f + \rho_m (1 - v_f - v_{\tilde{v}}) \\
E_x &= E_f V_f + E_m V_m \\
E_y &= \left[\frac{E_f \times E_m}{V_f E_m + V_m E_f}\right] \\
\gamma_{yz} &= \gamma_f V_f + \gamma_m (1 - V_f)\left[\frac{1 + \gamma_m - \frac{\gamma_{xy} E_m}{E_{xx}}}{1 - \gamma_m{}^2 + \frac{\gamma_m \gamma_{xy} E_m}{E_{xx}}}\right] \\
G_{xy} &= \frac{G_m G_f}{V_m G_f + V_f G_m} \quad \text{and} \quad G_{yz} = \frac{E_{yy}}{2(1+\gamma_{yz})}
\end{aligned}\right\} \quad (4)$$

Where $U_i$ : any mechanical property in functional form, the subscript c, f , m & $\tilde{v}$ stands for composite, fiber ,matrix an void respectively. V,ρ, γ, E, G, stands for volume fraction, density, poison's ratio, elastic modulus and shear modulus respectively.

The fiber reinforced in the beam is in woven form. The elastic constants of the woven fabric composite material are estimated by relating them to the properties of uni-directional composite material as using following relations:

$$\left.\begin{aligned}
E_{wx} &= \frac{E_x}{2}\left(\frac{E_x + (E_x + 2E_y) + (1 + 2\gamma_{xy}^2)E_y^2}{E_x\left(E_x + (1 - \gamma_{xy}^2)E_y\right) - \gamma_{xy}^2 E_y^2}\right) \\
\gamma_{wxy} &= \frac{4Ew_x}{E_x}\left(\frac{\gamma_{xy} E_y (E_x - \gamma_{xy}^2 E_y)}{E_x(E_x + 2E_y) + (1 + 2\gamma_{xy}^2)E_y^2}\right) \\
\gamma_{wxz} &= \frac{Ew_x}{E_x}\left(\frac{E_x(\gamma_{xy} + \gamma_{yz} + \gamma_{xy}\gamma_{yz}) + \gamma_{xy}^2 E_y}{E_x + (1 + 2\gamma_{xy})E_y}\right) \\
Ew_z &= \frac{(1 - \gamma_{yz}^2)E_x^2 + (1 + 2\gamma_{xy} + 2\gamma_{xy}\gamma_{yz})E_x E_y - \gamma_{xy}^2 E_y^2}{E_x E_y \left(E_x + (1 + 2\gamma_{xy})E_y\right)}
\end{aligned}\right\} \quad (5)$$

$Gw_{xy} = G_{xy}$ , $G_{wxz} = \frac{2G_{xy}E_y}{1+\gamma_{yz}}$

Where the subscript w stands for woven fabric composite

For the variation of fiber orientation, the stiffness of the composite component also varies. Hence the natural frequency will change accordingly.

**Table 1. Mechanical Property of Constituent Materials**

| Properties | Material | |
|---|---|---|
| | Glass fiber | Polyester resin |
| Elasticity modulus | 80GPa | 3.5GPa |
| Shear modulus | 30.3GPa | 1.26GPa |
| Density | 2600 kg/m$^3$ | 1200kg/m$^3$ |
| Poisson ratio | 0.32 | 0.38 |

## III PROBLEM FORMULATION

E-glass-polyester composite plates like beams are taken for consideration. The mechanical properties of the beam specimens taken for the study are calculated by using the relations for woven fabric composite as given by the relations (5) from the constituent material properties for matrix as presented in table-1. The beams are of same dimension considered with different end boundary conditions i.e. cantilever, clamped-clamped, clamped-simple supported and simple-simple supported boundary conditions. The geometric dimension of the beam for the study is taken as L=0.45m, T=0.007m and b =0.07m as shown in figure 2. The analysis is made for different fiber volume fraction varied from 0.3 to 0.6. The composite beam is composed of six number of ply layers. The orientations of upper three layers are exactly opposite to the lower three layers. This type of arrangement of plies can be done with even number of ply layers and are called as symmetric orientation. The general expression of the type of is expressed as ± ($\theta_1$-$\theta_2$-$\theta_3$). Six combinations of plies orientations with 15˚, 30˚and 45˚ are taken for the study. Also fiber layers with orientation ± (0˚-30˚-15˚ ± (0˚-15˚-30˚), and ± (0˚-0˚-0˚) are considered for the study. The beams were descretized using solid brick 8 node 185 elements that's each node has six degrees of freedom. The meshing of the beam is refined by taking the node length of 5mm for better result. To check the validity of the procedure the numerical example as in Goda et al. [8] is taken for the study and the result was presented in figure 3.

From the graph it can be observed that the result obtained from the finite element analysis by the help of ANSYS have closer value both to the experimental result and numerical analysis of the reference

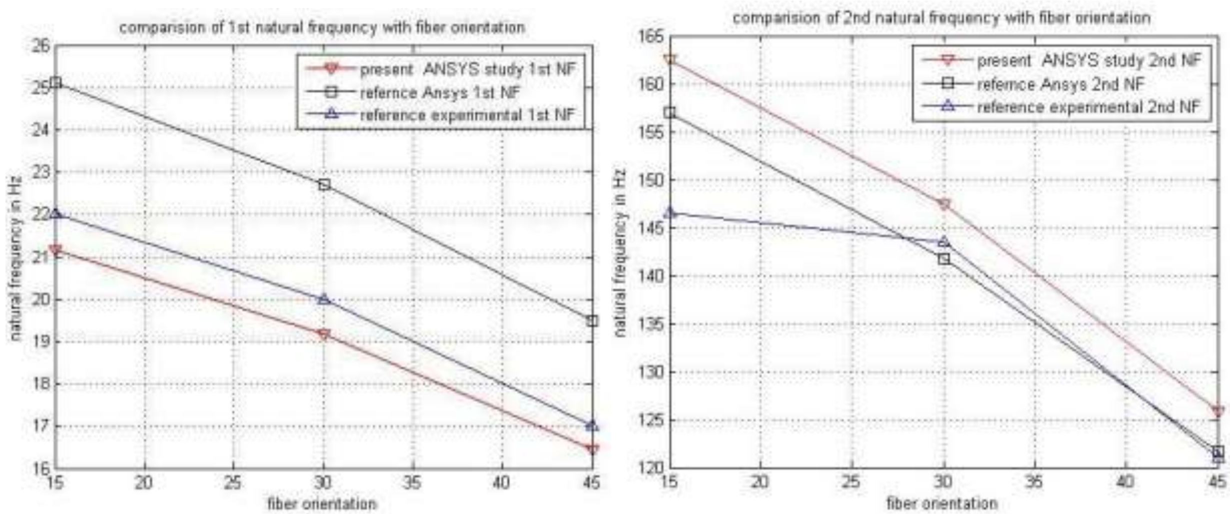


**Figure3: Comparison of natural frequencies for validation of the study**

## IV RESULT

The first five natural frequencies are obtained by using the commercial finite element analysis (FEA) software (ANSYS13.0). The first five in plane bending natural frequencies (NF) for the beam specimens are obtained from finite element analysis by the help of commercial finite element package ANSYS13.0. The variable parameters for the study are taken as outer fiber orientations, end conditions and volume fraction.

The variations of first five inplane bending natural frequencies obtained from FEA analysis with fiber volume fraction for different stacking sequences of the fiber layers for cantilever beams are presented through Figure 4(a) to Figure 4(e). the graph shows that the natural frequency increases with increase in fiber volume fraction. As the fiber is more stronger than matrix and provides strength to the composite material. So natural frequency increases .

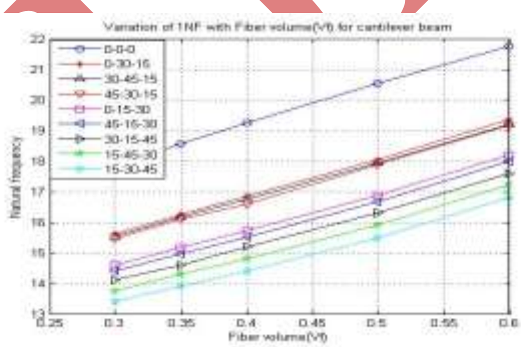


**Figure 4(a) variation of 1st NF of cantilever beam**

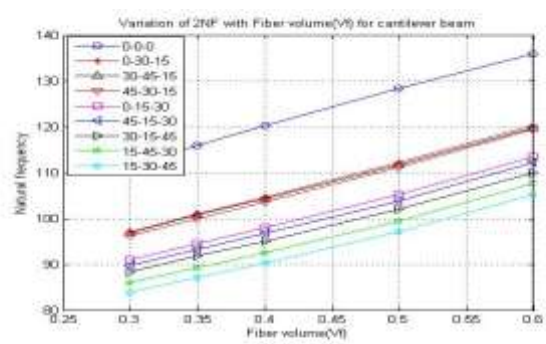


**Figure 4(b) variation of 2nd NF of cantilever**

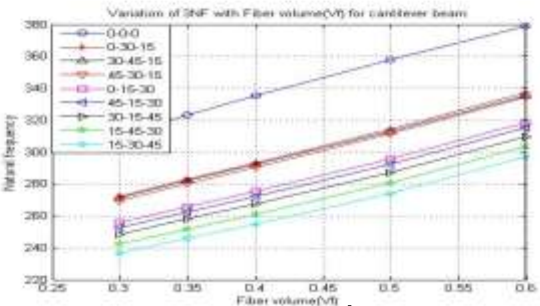


**Figure 4(c) variation of 3rd NF of**

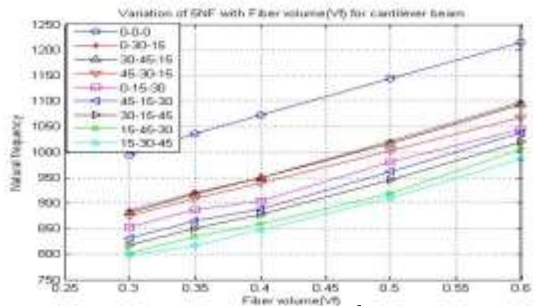


**Figure 4(d) variation of 4th NF of**

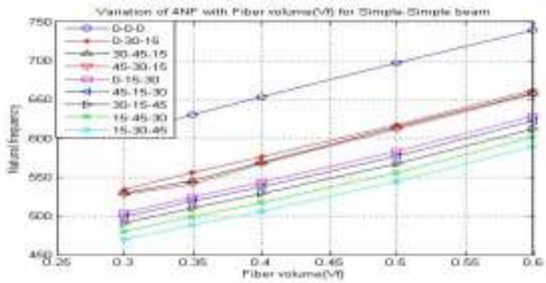


**Figure 4(e) variation of 5th NF of**

From the above graph it is clear that increase in volume fraction increases natural frequency for all of the orientations. The variations of natural frequency with fiber volume fraction for other boundary conditions of the beam are presented through figure-5 to figure-7.

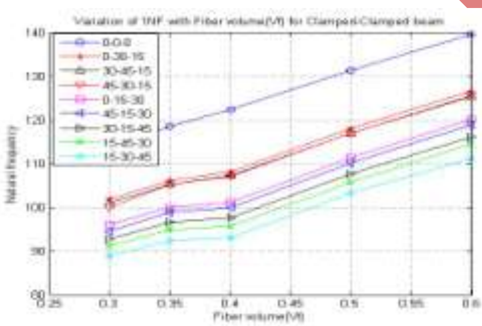


**Figure 5(a) variation of 1st NF of clamped-clamped beam**

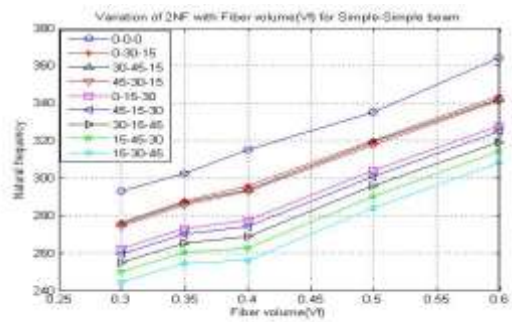


**Figure 5(b) variation of 2nd NF of clamped-clamped beam**

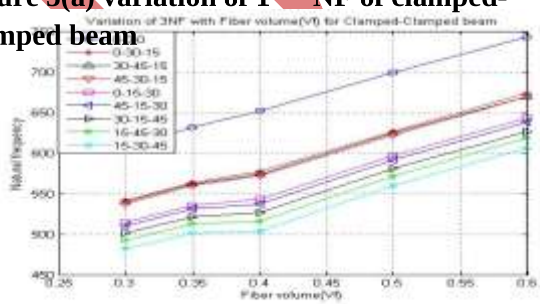


**Figure 5(c) variation of 3rd NF of clamped-clamped beam**

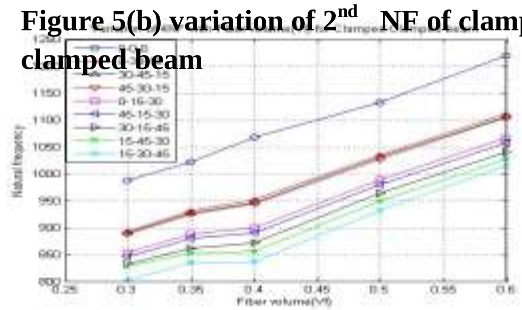


**Figure 5(d) variation of 4th NF of clamped-clamped beam**

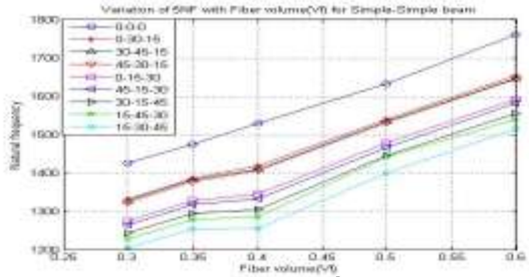

**Figure 5(e) variation of 5th NF of clamped-clamped beam**

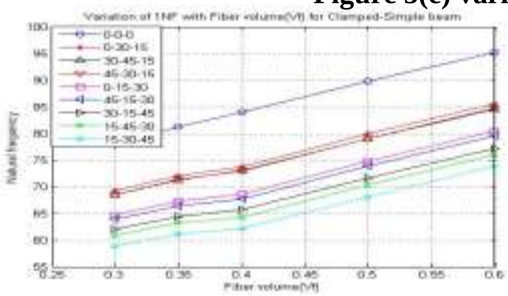

**Figure 6(a) variation of 1st NF of clamped-simple beam**

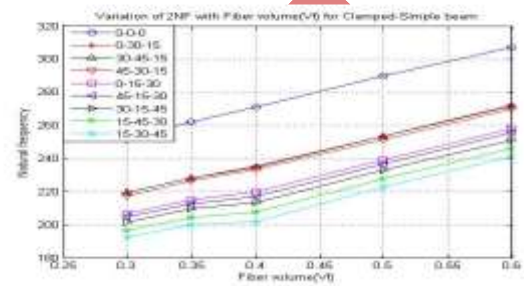

**Figure 6(b) variation of 2nd NF of clamped-simple beam**

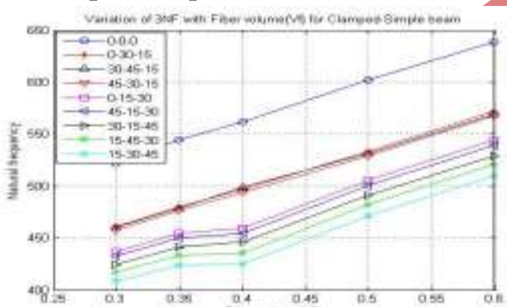

**Figure 6(c) variation of 3rd NF of clamped-simple beam**

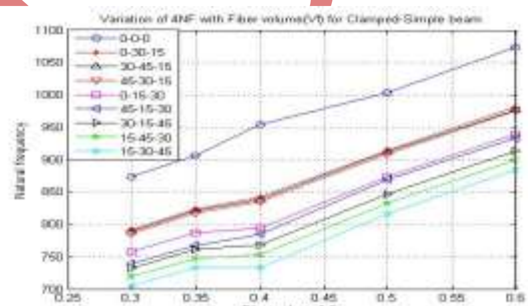

**Figure 6(d) variation of 4th NF of clamped-simple beam**

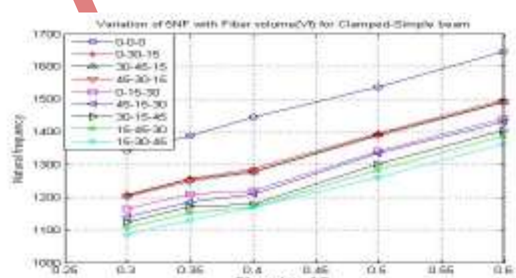

**Figure 6(e) variation of 5th NF of clamped-simple beam**

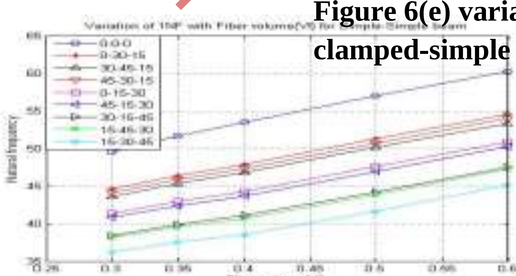

**Figure 7(a) variation of 1st NF of simple-simple beam**

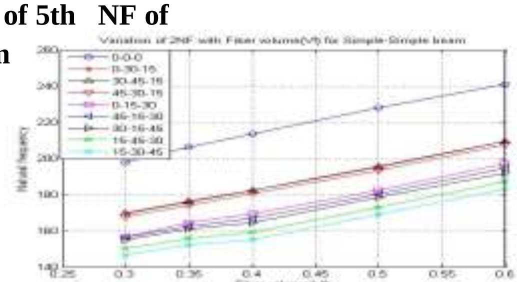

**Figure 7(b) variation of 2nd NF of simple-simple beam**

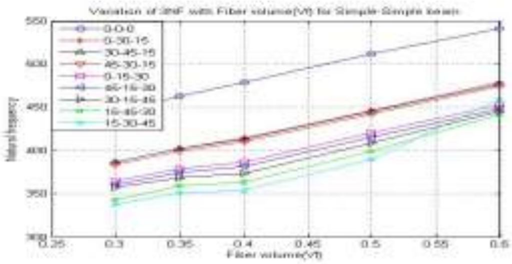

**Figure 7(c) variation of 3rd NF of simple-simple beam**

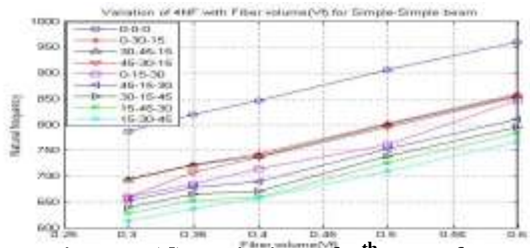

**Figure 7(d) variation of 4th NF of simple-simple beam**

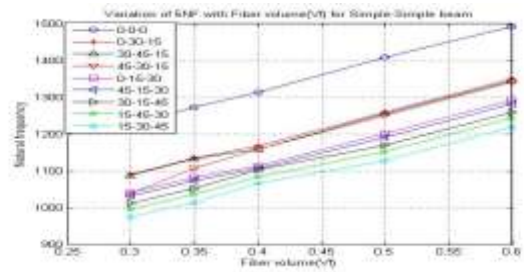

**Figure 7(e) variation of 5th NF of simple-**

From the above studies it is assumed that as the mechanical properties of the fiber is superior than the mechanical properties of the matrix , there is increase in natural frequency with increase in fiber volume fraction which in turn states that there is an increase in bending stiffness. So it is quite important to study the variation of modulus of elasticity and shear modulus with fiber volume fraction. Figure -8 shows the curve between the Elastic ratios and Fiber Volume fraction for Polyester-Glass Composite specimens. For the study fiber volume fraction is varied from 0.3 to 0.6 with 5% increment. It is observed that with increase in fiber volume fraction there is an increase in elastic ratios and it becomes maximum at $V_f$=0.5 and then starts to decrease thereafter as illustrated in the figure-8. So it can be said that the choice of volume fraction may be within 0.4 to 0.55 for best rest results for Polyester-Glass composite materials.

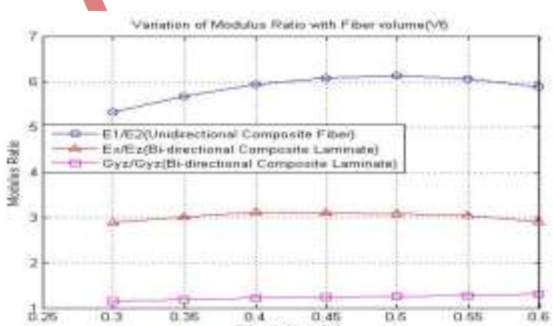


**Figure 8 variations of modulus ratios with**

## V CONCLUSION

Composite materials have wide range of engineering applications in various fields. The mechanical properties of composite beams depend upon the properties of fiber and matrix. It is also true that the fiber orientation and aspect ratio affects the mechanical properties like strength and rigidity of the material which in turn affects the natural frequency for same geometry and boundary condition. In this work effect of fiber volume fraction on elastic ratios and natural frequencies are studied. It has been found out that with change in fiber volume fraction there is change in natural frequency. It may be concluded that the choice of fiber volume fraction should be within 0.4 to 0.55 with 0.45 volume fraction having optimum result. Here Composite beams with ± (0˚-0˚-0˚) orientation is found to have highest value of natural frequency and ± (15˚-30˚-45˚) have lowest value of natural frequency for all of the configurations and boundary conditions as same as from the previous study [16, 17,18]. In future further study will be carried out in this field.

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