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INTERVAL - VALUED INTUITIONISTIC FUZZY ASSIGNMENT PROBLEM WITH REPLACEMENT BASED ON FUZZY AGGREGATION

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ABSTRACT

In this paper the theory of interval valued intuitionistic fuzzy sets (IVIFS) is used to propose an interval valued intuitionistic fuzzy assignment model involving the replacements. Under the fuzzy environment the individuals may provide the information conveniently with interval valued intuitionistic fuzzy numbers (IVIFN). Corporate competitiveness is heavily influenced by the professional (skilled) staff involved in the decision making. The job performance of a professional may correlate to the time taken to task. Two issues being analysed in this approach. The first one is to develop an ideal priority of the professional staff of the groups and the second one is to replace the staff where we need to avoid breakdowns. Replacement is made in the form of maximum intuitionistic fuzzy scoring by utilizing the intuitionistic fuzzy aggregation operator to aggregate the given intuitionistic fuzzy information and by the weight vector. The feasibility and applicability of the proposed method are illustrated with a numerical problem.

Keywords : Aggregative operators, Ideal priority, IVIF set, IVIF number.

I. INTRODUCTION

Atanassov and Gargov (1989) generalized the concept of instuitionistic fuzzy set (IFS) to interval - valued intuitionistic fuzzy set (IVIFS) and define some basic operational laws of IVIFS. In the corporate sector the assignment problem plays a vital role. Among researchers it has received a significant amount of attention. The costs are not known exactly in real world application. Employing fuzzy theory to model uncertainity in real problem it is assumed that the membership function of parameters are known. However in reality it is not always easy. In order to solve this problem, the best thing is to determine the uncertain as intervals. In some situations if an individual is not familiar with the problem, it is difficult to determine the exact values of the preference degrees. His views may be positive, negative and hesitative points. In this realistic situation intuitionistic fuzzy set is the useful tool to express.

In most of the situation it is difficult for a single man to consider all their important aspects of some decision problem. So corporate employ group of people to do a single job. This gives a group setting. It requires to

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aggregate all individual decision into a collective decision. Since the individuals in the groups may have different capabilities it is necessary to use weighted aggregation. In the present complex socio economic environment and the insufficient knowledge of the problem, under the fuzzy environment, individuals in the group may provide their information over alternatives with interval valued intuitionistic fuzzy number (IVIFN).

Chen (1985) introduced a fuzzy assignment model that considers all individuals have same skills. Huang and Zhang (2006) proposed a mathematical model for the fuzzy assignment problem with restriction on qualification. Mukherjee and Basu (2011) proposed intuitionistic fuzzy assignment problem by using similarity measures and score functions. Xu (2007) defined the concept of interval - valued intuitionistic fuzzy number. Xu and Chen (2007) define an interval-valued intuitionistic fuzzy ordered weighted averaging operator and an interval-valued intuitionistic hybrid averaging operator. Lin and Wen (2004) concentrate on the assignment problem where costs are not deterministic numbers but imprecise ones. Gaurav and Bajaj (2014) proposed interval-valued intuitionistic fuzzy assignment problem by using similarity measure and score functions.

II. Preliminaries On Interval Valued Intuitionistic Fuzzy Sets

This section presents the basic concepts related to Interval-Valued Intuitionistic Fuzzy Set, which was originally introduced by Atanassov and Gargov (1989).

2.1. Interval-Valued Intuitionistic Fuzzy Set (IVIFS):

Let X be a fixed set. Then $\tilde{A} = \{\langle x, \tilde{\mu}_{\tilde{A}}(x), \tilde{\vartheta}_{\tilde{A}}(x) \rangle | x \in X\}$ is called an interval- valued intuitionistic fuzzy set (IVIFS), where $\tilde{\mu}_{\tilde{A}}(x) \subset [0,1]$ and $\tilde{\vartheta}_{\tilde{A}}(x) \subset [0,1], \; x \in X$, with the condition:

$$sup\,\tilde{\mu}_{\tilde{A}}(x) + sup\,\tilde{\vartheta}_{\tilde{A}}(x) \leq 1, \quad x \in X.$$

Clearly, if $inf\;\tilde{\mu}_{\tilde{A}}(x) = sup\;\tilde{\mu}_{\tilde{A}}(x)$ and $inf\;\tilde{\vartheta}_{\tilde{A}}(x) = sup\;\tilde{\vartheta}_{\tilde{A}}(x)$, then IVIFS $\tilde{A}$ reduces to a traditional IFS.

2.2 Ranking Of Interval Valued Intuitionistic Fuzzy Number :

Definition 1

Let $\tilde{\alpha} = ([a,b],[c,d])$ be an IVIFN. Then we call

$$s(\tilde{\alpha}) = \frac{1}{2}\,[(a-c)+(b-d)], \qquad (1)$$

the score of $\tilde{\alpha}$, s is the score function of $\tilde{\alpha}$, $s(\tilde{\alpha}) \in [-1,1]$. Clearly the greater the $s(\tilde{\alpha})$, the larger the $\tilde{\alpha}$. In particular, if $s(\tilde{\alpha}) = 1$, then $\tilde{\alpha}$ is the largest IVIFN: ([1, 1] , [0, 0]); If $s(\tilde{\alpha}) = -1$, then $\tilde{\alpha}$ is the smallest IVIFN: ([0, 0], [1, 1]).

The accuracy function of an IVIFN $\tilde{\alpha}$ is defined as

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$h(\tilde{\alpha}) = \frac{1}{2}\ (a+b+c+d)$, where $h(\tilde{\alpha}) \in [0,1]$. (2)

Definition 2

Let $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ be any two IVIFNs. Then

if $s(\tilde{\alpha}_1) < s(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1 < \tilde{\alpha}_2$

if $s(\tilde{\alpha}_1) = s(\tilde{\alpha}_2)$ and

if $h(\tilde{\alpha}_1) = h(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1 = \tilde{\alpha}_2$

if $h(\tilde{\alpha}_1) < h(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1 < \tilde{\alpha}_2$

if $h(\tilde{\alpha}_1) > h(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1 > \tilde{\alpha}_2$.

2.3 Operational Laws Of Interval-Valued Intuitionistic Fuzzy Number (Xu, 2007) :

Let $\tilde{\alpha} = ([a,b],[c,d])$, $\tilde{\alpha}_1 = ([a_1,b_1],[c_1,d_1])$ and $\tilde{\alpha}_2 = ([a_2,b_2],[c_2,d_2])$ be IVIFNs. Then

(1) $\overline{\tilde{\alpha}} = ([c,d],[a,b])$ where $\overline{\tilde{\alpha}}$ is the complement of $\tilde{\alpha}$

(2) $\tilde{\alpha}_1 \wedge \tilde{\alpha}_2 = ([\min\{a_1,a_2\},\min\{b_1,b_2\}],[\max\{c_1,c_2\},\max\{d_1,d_2\}]);$

(3) $\tilde{\alpha}_1 \vee \tilde{\alpha}_2 = ([\max\{a_1,a_2\},\max\{b_1,b_2\}],[\min\{c_1,c_2\},\min\{d_1,d_2\}]);$

(4) $\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 =([a_1+a_2-a_1a_2, b_1+b_2-b_1b_2],\ [c_1c_2, d_1d_2]);$

(5) $\tilde{\alpha}_1 \otimes \tilde{\alpha}_2 =([a_1a_2, b_1b_2],\ [c_1+c_2-c_1c_2, d_1+d_2-d_1d_2]);$

(6) $\lambda\tilde{\alpha} = \left([1-(1-a)^\lambda, 1-(1-b)^\lambda],[c^\lambda, d^\lambda]\right), \lambda > 0;$

(7) $\tilde{\alpha}^\lambda = \left([a^\lambda, b^\lambda],[1-(1-c)^\lambda, 1-(1-d)^\lambda]\right), \lambda > 0.$

Definition 3

Let ω be the weight vector. Weight is the ratio of cost in the ij^th cell to the total cost for each job J_i. i.e.,

$$\omega_{ij} = \frac{c_{ij}}{\sum_{j=1}^{n} c_{ij}},\quad i = 1,2,\dots,n. \qquad (3)$$

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Also $\sum_{j=1}^{n} \omega_{ij} = 1$ for each i. We define $\omega_1 = (\omega_{11}, \omega_{12}, \dots, \omega_{1n})$, $\omega_2 = (\omega_{21}, \omega_{22}, \dots, \omega_{2n})$, … ,

$\omega_n = (\omega_{n1}, \omega_{n2}, \dots, \omega_{nn})$.

2.4 Interval -Valued Intuitionistic Fuzzy Hybrid Averaging Operator (Xu And Chen 2007) :

Let $\tilde{\Theta}$ be the set of all IVIFNs. An interval-valued intuitionistic fuzzy hybrid averaging (IIFHA) operator is a mapping IIFHA : $\tilde{\Theta}^n \to \tilde{\Theta}$. Suppose

$$IIFHA_{\omega,\omega}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \omega_1 \dot{\tilde{\alpha}}_{\sigma(1)} \oplus \omega_2 \dot{\tilde{\alpha}}_{\sigma(2)} \oplus \dots \oplus \omega_n \dot{\tilde{\alpha}}_{\sigma(n)} \quad (4)$$

where $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weighting vector associated with the IIFHA function, with $\omega_j \in [0,1]$ $(j = 1,2, \dots, n)$ and $\sum_{j=1}^{n} \omega_j = 1$. $\dot{\tilde{\alpha}}_{\sigma(j)}$ is the j^th largest of the weighted IVIFNs $\dot{\tilde{\alpha}}_i$ $(i = 1,2, \dots, n)$, here $\dot{\tilde{\alpha}}_i = n\omega_i \tilde{\alpha}_i$ $(i = 1,2, \dots, n)$, $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weight vector of a collection of the IVIFNs $\tilde{\alpha}_i$ ($i = 1,2, \dots, n$), with $\omega_j \in [0,1]$ $(j = 1,2, \dots, n)$ and $\sum_{j=1}^{n} \omega_j = 1$, and n is the balancing coefficient.

Theorem 1 (Xu and Chen 2007)

Assume that $\dot{\tilde{\alpha}}_i = ([\dot{a}_i, \dot{b}_i], [\dot{c}_i, \dot{d}_i])$ $i = 1,2, \dots, n$ and $\dot{\tilde{\alpha}}_{\sigma(j)} = ([\dot{a}_{\sigma(j)}, \dot{b}_{\sigma(j)}], [\dot{c}_{\sigma(j)}, \dot{d}_{\sigma(j)}])$ $j = 1,2, \dots, n$.
Then the aggregated value by using equation (4) is an IVIFN, and

$$IIFHA_{\omega,\omega}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) =$$

$$\left([1 - \prod_{j=1}^{n}(1 - \dot{a}_{\sigma(j)})^{\omega_j}, 1 - \prod_{j=1}^{n}(1 - \dot{b}_{\sigma(j)})^{\omega_j}], [\prod_{j=1}^{n} \dot{c}_{\sigma(j)}{}^{\omega_j}, \prod_{j=1}^{n} \dot{d}_{\sigma(j)}{}^{\omega_j}]\right) \quad (5)$$

III. Multi-Attribute Decision Making Problem

Let $P = \{P_1, P_2, \dots, P_n\}$ be a finite set of alternatives, $Q = \{Q_1, Q_2, \dots, Q_m\}$ a set of attributes, and $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ the weight vector of attributes, where

$\omega_j \in [0,1]$ $(j = 1,2, \dots, m)$ and $\sum_{j=1}^{n} \omega_j = 1$. Suppose that the characteristic information on the alternatives P_i $(i = 1,2, \dots, n)$ are represented by the IVIFNs:

$P_i = \{\langle Q_j, \tilde{\mu}_{P_i}(Q_j), \tilde{\vartheta}_{P_i}(Q_j)\rangle | Q_j \epsilon Q\}, \quad i = 1,2,\ldots,n$ (6)

where $\tilde{\mu}_{P_i}(Q_j)$ indicates the degree that the alternative P_i satisfies the attributes Q_j, $\tilde{\vartheta}_{P_i}(Q_j)$ indicates the degree that the alternative P_i does not satisfy the attribute Q_j. Here $\tilde{\mu}_{P_i}(Q_j)$ and $\tilde{\vartheta}_{P_i}(Q_j)$ are given in the value ranges, i.e., interval numbers, and $\tilde{\mu}_{P_i}(Q_j) \subset [0,1]$, $\tilde{\vartheta}_{P_i}(Q_j) \subset [0,1]$, $sup\,\tilde{\mu}_{P_i}(Q_j) + \sup\, \tilde{\vartheta}_{P_i}(Q_j) \le 1$ (7)

Let $\tilde{\mu}_{P_i}(Q_j) = \tilde{\mu}_{ij} = [\tilde{\mu}_{ij}{}^{L}, \tilde{\mu}_{ij}{}^{U}]$ and $\tilde{\vartheta}_{P_i}(Q_j) = \tilde{\vartheta}_{ij} = [\tilde{\vartheta}_{ij}{}^{L}, \tilde{\vartheta}_{ij}{}^{U}]$, $(i = 1,2,\ldots,n);(j = 1,2,\ldots m)$.

Consequently, we can get the interval-valued intuitionistic fuzzy decision matrix $\tilde{R}' = (\tilde{r}'_{ij})_{n\times m}$,

$\tilde{r}'_{ij} = (\tilde{\mu}'_{ij}, \tilde{\vartheta}'_{ij}) = ([\tilde{\mu}'_{ij}{}^{L}, \mu'_{ij}{}^{U}], [\tilde{\vartheta}'_{ij}{}^{L}, \vartheta'_{ij}{}^{U}])$. In general, there are two types of attributes, that is benefit attributes and cost attributes. We can normalize $\tilde{R}' = (\tilde{r}'_{ij})_{n\times m}$ into the interval-valued intuitionistic fuzzy decision matrix $\tilde{R} = (\tilde{r}_{ij})_{n\times m}$,

where $\tilde{r}_{ij} = (\tilde{\mu}_{ij}, \tilde{\vartheta}_{ij}) = \left([\tilde{\mu}_{ij}{}^{L}, \tilde{\mu}_{ij}{}^{U}], [\tilde{\vartheta}_{ij}{}^{L}, \tilde{\vartheta}_{ij}{}^{U}]\right)$

$$= \begin{cases} \tilde{r}'_{ij}\ , & for\ benifit\ attribute\ Q_j\ , \\ \bar{\tilde{r}}'_{ij}\ , & for\ cost\ attribute\ Q_j . \end{cases} \quad (8)$$

where $\bar{\tilde{r}}'_{ij}$ is the complement of $\tilde{r}'_{ij}$, $(i = 1,2,\ldots,n);(j = 1,2,\ldots m)$.

IV. Mathematical Model Of Interval Valued Intuitionistic Fuzzy Assignment Problem

An assignment problem is a special type of transporation problem which can be stated in the form of $n \times n$ cost matrix $[\tilde{c}_{ij}]$ of interval-valued intuitionistic fuzzy numbers as follows:

Table 1 : Cost matrix of an IVIF assignment problem

Job / Group	1	2	...	n
1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	...	$\tilde{c}_{1n}$

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2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	...	$\tilde{c}_{2n}$
⋮	⋮	⋮	⋮	⋮
n	$\tilde{c}_{n1}$	$\tilde{c}_{n2}$	...	$\tilde{c}_{nn}$

The objective is to assign a number of origins to an equal number of destinations at a minimum cost or maximum profit. Each job must be done by exactly one group and one group can do, at most one job. Mathematically assignment problem can be denoted as

$$\text{Min}\, Z = \sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,2,\dots,n \tag{9}$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,2,\dots,n \tag{10}$$

where x_{ij} is the decision variable defined as

$$x_{ij} = \begin{cases} 1, \text{if the } i^{th} \text{ group is assigned to the } j^{th} \text{ job,} \\ 0, \text{otherwise}; \quad \text{where } i,j = 1,2,\dots,n. \end{cases}$$

The cost of a group i doing the job j is considered as an interval-valued intuitionistic fuzzy number $\tilde{c}_{ij} = (\tilde{\mu}_{ij}, \tilde{\vartheta}_{ij}) = \left([\tilde{\mu}_{ij}{}^{L}, \tilde{\mu}_{ij}{}^{U}], [\tilde{\vartheta}_{ij}{}^{L}, \tilde{\vartheta}_{ij}{}^{U}]\right), i,j = 1,2,\dots,n$ where $\tilde{\mu}_{ij}{}^{L}, \tilde{\mu}_{ij}{}^{U}$ denotes the degree of acceptance and $\tilde{\vartheta}_{ij}{}^{L}, \tilde{\vartheta}_{ij}{}^{U}$ denotes the degree of rejection.

As our objective is to minimize the cost and maximize the profit, we should go for maximize the acceptance degree $\tilde{\mu}_{ij}$. Then the objective function becomes a multi-objective function as

$Max\ z_1 = \sum_{i=1}^{n}\sum_{j=1}^{n}\left([\tilde{\mu}_{ij}{}^{L}, \tilde{\mu}_{ij}{}^{U}]\right)x_{ij}$ with condition $\tilde{\mu}_{ij}{}^{U} + \tilde{\vartheta}_{ij}{}^{U} \le 1$ and

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$Min\ z_2 = \sum_{i=1}^{n}\left(\left[\tilde{\vartheta}_{ij}^{\ L}, \tilde{\vartheta}_{ij}^{\ U}\right]\right) x_{ij}$ with condition $\tilde{\mu}_{ij}^{\ U} + \tilde{\vartheta}_{ij}^{\ U} \leq 1$

subject to $\left(\tilde{\mu}_{ij}^{\ U} + \tilde{\vartheta}_{ij}^{\ U} - 1\right) x_{ij} \leq 0,$ (11)

$(\tilde{\mu}_{ij}^{\ U})\, x_{ij} \geq (\tilde{\vartheta}_{ij}^{\ U})\, x_{ij}$ (12)

$(\tilde{\vartheta}_{ij}^{\ U})\, x_{ij} \geq 0$ (13)

Thus the model becomes

$\max Z = \sum_{i=1}^{n} \sum_{j=1}^{n} (\tilde{\mu}_{ij}^{\ U} - \tilde{\vartheta}_{ij}^{\ U}) x_{ij.}$

subject to the conditions (9), (10), (11), (12) and (13).

V. SOLUTION PROCEDURE

Step 1: Identify the teams with members and jobs. Fix the number of persons (alternatives) in a team and fix characteristics (attributes) to develop ideal priorities to each job.

Step 2: Represent attributes of the alternatives by the IVIFNs in an interval-valued intuitionistic fuzzy decision matrix $\tilde{R}' = (\tilde{r}'_{ij})_{n\times m}$

Step 3: Transform the attribute value of cost type of each job into attribute values of benefit type using equation (8) and represent the transformed matrix as $\tilde{R} = (\tilde{r}_{ij})_{n\times m}$

Step 4: Weight all the alternative values $\tilde{r}_{ij}$ by the weight vector ω_j (as in equation (3)) of the attributes and multiply these values by the coefficient values n and then get the weighted attribute values $n\omega_j \tilde{r}_{ij}$ and represent in an interval-valued intuitionistic fuzzy decision matrix $\dot{\tilde{R}} = (n\omega_j \tilde{r}_{ij})_{n\times m}$

Step 5: Utilize the IIFHA operator $\dot{\tilde{r}}_i = IIFHA_{\omega,\omega}(\tilde{r}_{i1}, \tilde{r}_{i2}, \dots, \tilde{r}_{im}), i = 1, 2, \dots, n$ calculate the aggregated values of $\dot{\tilde{r}}_i$.

Step 6: Utilize equation (1) calculate the scores of $\dot{\tilde{r}}_i$ ($i = 1,2,\dots n$) and get the ideal priorities among persons of a team to each job according to the heirarchical score values. If two scores are equal we may use the accuracy function given in equation (2) to compare the two IVIFNs.

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Step 7: If there is no breakdown find the maximum of acceptance and minimum of rejection of the cost to do each job by the persons of the teams. Assign these IVIFNs representing the cost in a matrix $\tilde{R}_1$ to a particular team to do a particular job.

Step 8: Find the score values of the entries of the matrix $\tilde{R}_1$ and represent in a matrix $\tilde{R}_2$. Using Hungarian method or any other software find the assignment.

Step 9: If there exist a break down in a team with a particular person then the assignment will be affected. Replace the person corresponding to the break down from the pool whose priority is higher in the ideal priority of the pool.

Step 10: Find the score values of the matrix $\tilde{R}_1$ and represent in a matrix $\tilde{R}_2$. Using Hungarian method find the assignment.

VI. ILLUSTRATIVE EXAMPLE

Let us consider five teams (alternatives) $Y_i\ (i = 1,2,\dots,5)$ to do five jobs. Each team consists of four skilled persons. By considering the following five attributes to decide the priorities of the persons in the team: w_1: capacity, w_2: performance, w_3: cost to do job, w_4: time management and w_5: Experience. Assume that the characteristic information of the alternatives $Y_i\ (i = 1,2,\dots,5)$ and the attribute value information corresponding to the person of the teams are represented by the IVIFNs.

Team Y_1

a) Let us consider team Y_1 correspondings to job J_1 and it has four alternatives $P_i\,(i = 1,2,3,4)$ and five attributes $w_j\ (j = 1,2,\dots 5)$. Assume that the characteristic information of the alternatives $P_i\ (i = 1,2,3,4)$ are represented by the IVIFNs, as shown in the interval-valued intuitionistic fuzzy decision matrix $\tilde{R}' = (\tilde{r}'_{ij})_{4\times 5}$

Table 2: Interval-valued intuitionistic fuzzy decision matrix $\tilde{R}'$

	w_1	w_2	J_1	w_3	w_4
P_1	([0.5,0.6].[0.2,0.4])	([0.5,0.8],[0.1,0.2])	([0.2,0.3],[0.4,0.7])	([0.4,0.5],[0.3,0.5])	([0.3,0.6],[0.2,0.4])
P_2	([0.7,0.8],[0.1,0.2])	([0.6,0.7],[0.1,0.2])	([0.1,0.3],[0.5,0.7])	([0.6,0.8],[0.1,0.2])	([0.4,0.1],[0.2,0.3])

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P_3	([0.2,0.4],[0.1,0.2])	([0.4,0.5],[0.2,0.4])	([0.0,0.1],[0.5,0.8])	([0.4,0.6],[0.2,0.4])	([0.5,0.6],[0.3,0.2])
P_4	([0.5,0.7],[0.1,0.3])	([0.4,0.7],[0.2,0.3])	([0.2,0.2],[0.4,0.5])	([0.7,0.8],[0.1,0.2])	([0.4,0.5],[0.1,0.2])

J_1: ([max (0.2, 0.1, 0, 0.2), (0.3, 0.3, 0.1, 0.2)], [min (0.4, 0.5, 0.5, 0.4), (0.7, 0.7, 0.8, 0.5)])

$i.e., \ J_1: ([0.2, 0.3], [0.4, 0.5])$

Transform the attribute values of cost type in the attribute values of benefit type by using equation (8). Then

$\tilde{R}' = (\tilde{r}'_{ij})_{4\times5}$ is transformed into $\tilde{R} = (\tilde{r}_{ij})_{4\times5}$.

Table 3: Interval-valued Intuitionistic fuzzy decision matrix $\tilde{R}$

	w_1	w_2	J_1	w_3	w_4
P_1	([0.5,0.6],[0.2,0.4])	([0.5,0.8],[0.1,0.2])	([0.4,0.7], [0.2,0.3])	([0.4,0.5],[0.3,0.5])	([0.3,0.6],[0.2,0.4])
P_2	([0.7,0.8],[0.1,0.2])	([0.6,0.7],[0.1,0.2])	([0.5,0.7], [0.1,0.3])	([0.6,0.8],[0.1,0.2])	([0.4,0.1],[0.2,0.3])
P_3	([0.2,0.4],[0.1,0.2])	([0.4,0.6],[0.2,0.4])	([0.5,0.8], [0.0,0.1])	([0.4,0.6],[0.2,0.4])	([0.5,0.6],[0.3,0.2])
P_4	([0.5,0.7],[0.1,0.3])	([0.4,0.7],[0.2,0.3])	([0.4,0.5], [0.2,0.2])	([0.7,0.8],[0.1,0.2])	([0.4,0.5],[0.1,0.2])

To get the ideal priorities the following steps are followed:

First weight all the attribute values $\tilde{r}_{ij}$ $(i = 1,2,3,4;\ j = 1,2,\ldots,5)$ by weight vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ of the attributes w_j $(j = 1,2,\ldots,5)$ and multiply these values by the balancing coefficient $m = 5$, and then get the weighted attribute values $5\omega_j\tilde{r}_{ij}$ $(i = 1,2,3,4;\ j = 1,2,\ldots,5)$, as listed in the weighted interval-valued intuitionistic fuzzy decision matrix $\dot{\tilde{R}} = (5\omega_j\tilde{r}_{ij})_{4\times5}$. Utilizing definition 3 calculate the weight vector of $\tilde{R}$.

$\omega_1 = (0.2, 0.4, 0.24, 0.04, 0.12), \omega_2 = (0.293, 0.244, 0.195, 0.268, 0),$

$\omega_3 = (0.107, 0.107, 0.429, 0.143, 0.214), \omega_4 = (0.216, 0.162, 0.136, 0.324, 0.162).$

From table 3 we utilize the operational laws (4) and (6) in section 2.3 to get weighted IFNs:

$\dot{\tilde{r}}_{11} = ([(1-(1-0.5)^{5\times0.2}, (1-(1-0.6)^{5\times0.2}], [0.2^{5\times0.2}, 0.4^{5\times0.2}]$ = ([0.5, 0.6], [0.2, 0.4]),

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$\hat{r}_{12} = ([(1-(1-0.5)^{5\times0.4}, (1-(1-0.8)^{5\times0.4}], [0.1^{5\times0.4}, 0.2^{5\times0.4}])$ = ([0.75, 0.96], [0.01, 0.04]),

$\hat{r}_{13} = ([(1-(1-0.4)^{5\times0.24}, (1-(1-0.7)^{5\times0.24}], [0.2^{5\times0.24}, 0.3^{5\times0.24}]$ = ([0.458, 0.764], [0.145, 0.236])

$\hat{r}_{14} = ([(1-(1-0.4)^{5\times0.04}, (1-(1-0.5)^{5\times0.04}], [0.3^{5\times0.04}, 0.5^{5\times0.04}]$ = ([0.097, 0.129], [0.786, 0.871]),

$\hat{r}_{15} = ([(1-(1-0.3)^{5\times0.12}, (1-(1-0.6)^{5\times0.12}], [0.2^{5\times0.12}, 0.4^{5\times0.12}]$ = ([0.193, 0.423], [0.381, 0.9]).

Similarly $\hat{r}_{2j}$, $\hat{r}_{3j}$ *and* $\hat{r}_{4j}$ *where* $j = 1,2,\ldots,5$.

Table 4: The weighted interval-valued intuitionistic fuzzy decision matrix $\hat{R}$

	w_1	w_2	l_1	w_3	w_4
P_1	([0.5, 0.6], [0.2, 0.4])	([0.75, 0.96], [0.01, 0.04])	([0.458, 0.764], [0.145 ,0.236])	([0.097, 0.129], [0.786, 0.871])	([0.193, 0.423], [0.381, 0.577])
P_2	([0.829, 0.905], [0.034, 0.095])	([0.673, 0.77], [0.06, 0.14])	([0.491, 0.691], [0.106, 0.309])	([0.707, 0.884], [0.046, 0.116])	([0, 0],[1, 1])
P_3	([0.113, 0.239], [0.292, 0.423])	([0.239, 0.31], [0.423, 0.612])	([0.774, 0.968], [0, 0.007])	([0.306, 0.481], [0.316, 0.519])	([0.524, 0.625], [0.276, 0.179])
P_4	([0.527, 0.728], [0.083, 0.272])	([0.339, 0.623], [0.272, 0.377])	([0.293, 0.376], [0.335, 0.335])	([0.858, 0.926], [0.024, 0.074])	([0.339, 0.43], [0.155, 0.272])

The scores of $\hat{r}_{ij}$ $(i = 1,2,3,4)$ *and* $(j = 1,2,\ldots,5)$ by equation (1) *are*

s($\hat{r}_{11}) = \frac{1}{2}\times(0.5-0.2+0.6-0.4) = 0.25$,

s($\hat{r}_{12}) = \frac{1}{2}\times(0.75-0.01+0.96-0.04) = 0.83$,

s($\hat{r}_{13}) = \frac{1}{2}\times(0.458-0.145+0.764-0.236) = 0.4205$,

s($\hat{r}_{14}) = \frac{1}{2}\times(0.097-0.786+0.129-0.871) = -0.7155$,

s($\hat{r}_{15}) = \frac{1}{2}\times(0.193-0.381+0.423-0.577) = -0.171$.

since s($\hat{r}_{12}) > s(\hat{r}_{13}) > s(\hat{r}_{11}) > s(\hat{r}_{15}) > s(\hat{r}_{14})$ we have

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$\dot{\alpha}_{\sigma(1)} = ([0.75, 0.96], [0.01, 0.04]$, $\dot{\alpha}_{\sigma(2)} = ([0.458, 0.764], [0.145, 0.236])$,

$\dot{\alpha}_{\sigma(3)} = ([0.5, 0.6], [0.2, 0.4])$, $\dot{\alpha}_{\sigma(4)} = ([0.193, 0.423], [0.381, 0.577])$,

$\dot{\alpha}_{\sigma(5)} = ([0.097, 0.129], [0.786, 0.871])$.

Similarly we can calculate the values of other rows.

Let $\omega = (0.112, 0.236, 0.304, 0.236, 0.112)^T$ be its weighting vector as derived by the normal distribution based method of Xu (2005). Utilize the IIFHA operator (4) derive the overall attribute values $\dot{r}_i$ $(i = 1,2,3,4)$ of alternatives P_i $(i = 1,2,3,4)$

$$\dot{r}_1 = (1 - (1 - 0.75)^{0.112} \times (1 - 0.458)^{0.236} \times (1 - 0.5)^{0.304} \times (1 - 0.193)^{0.236} \times (1 - 0.097)^{0.112}, (1 - (1 - 0.96)^{0.112} \times (1 - 0.764)^{0.236} \times (1 - 0.6)^{0.304} \times (1 - 0.423)^{0.236} \times (1 - 0.129)^{0.112}, \quad 0.01^{0.112} \times 0.145^{0.236} \times 0.2^{0.304} \times 0.381^{0.236} \times 0.786^{0.112}), \quad 0.04^{0.112} \times 0.236^{0.236} \times 0.4^{0.304} \times 0.577^{0.236} \times 0.871^{0.112})$$

$$= ([0.436, 0.675], [0.18, 0.325])$$

We obtained

$\dot{r}_1 = ([0.436, 0.675], [0.18, 0.325])$, $\dot{r}_2 = ([0.63, 0.786], [0.081, 0.19])$,

$\dot{r}_3 = ([0.401, 0.602], [0, 0.242])$, $\dot{r}_4 = ([0.482, 0.651], [0.135, 0.26])$

The scores of $\dot{r}_i$ $(i = 1,2,3,4)$ by equation (1) are

$s(\dot{r}_1) = \frac{1}{2} \times (0.436 - 0.18 + 0.675 - 0.325) = 0.303$,

$s(\dot{r}_2) = \frac{1}{2} \times (0.63 - 0.081 + 0.786 - 0.19) = 0.5725$,

$s(\dot{r}_3) = \frac{1}{2} \times (0.401 - 0 + 0.602 - 0.242) = 0.3805$,

$s(\dot{r}_4) = \frac{1}{2} \times (0.482 - 0.135 + 0.651 - 0.26) = 0.369$.

Since $s(\dot{r}_2) > s(\dot{r}_3) > s(\dot{r}_4) > s(\dot{r}_1)$, we represent the ideal priorities of the persons of team Y_1 corresponding to job J_1 as $P_2 > P_3 > P_4 > P_1$.

b) Consider the cost values of the persons of the team Y_1 to the job J_2 as $P_1:([0.2, 0.3], [0.1,0.7])$, $P_2:([0, 0.2], [0.8,0.7])$, $P_3:([0.2, 0.1], [0.5,0.8])$ and $P_4:([0.4, 0.2], [0.3,0.5])$ and the remaining values of attributes are as in Table 2.The ideal priorities of the persons of team Y_1 corresponding to job J_2 represented by $P_2 > P_4 > P_3 > P_1$.

c) Consider the cost values of the persons of the team Y_1 to the job J_3 as $P_1:([0.2, 0.3], [0.6,0.7])$, $P_2:([0.1, 0.5], [0.3,0.4])$, $P_3:([0.4, 0.1], [0.2,0.6])$ and $P_4:([0.3, 0.4], [0.4,0.6])$ and the remaining values of attributes are as in Table 2. The ideal priorities of the persons of team Y_1 corresponding to job J_3 represented by $P_2 > P_4 > P_1 > P_3$.

d) Consider the cost values of the persons of the team Y_1 to the job J_4 as $P_1:([0.1, 0.2], [0.3,0.2])$, $P_2:([0.1, 0.1], [0.8,0.9])$, $P_3:([0.1, 0.2], [0.6,0.2])$ and $P_4:([0.4, 0.2], [0.3,0.4])$ and the remaining values of attributes are as in Table 2. The ideal priorities of the persons of team Y_1 corresponding to job J_4 represented by $P_2 > P_4 > P_3 > P_1$.

e) Consider the cost values of the persons of the team Y_1 to the job J_5 as $P_1:([0.3, 0.4], [0.5,0.6])$, $P_2:([0.5, 0.2], [0.3, 0.4])$, $P_3:([0.4, 0.1], [0.2, 0.3])$ and $P_4:([0.4, 0.2], [0.3,0.5])$ and the remaining values of attributes are as in Table 2. The ideal priorities of the persons of team Y_1 corresponding to job J_5 represented by $P_2 > P_4 > P_3 > P_1$.

Team Y_2

a) Let us consider team Y_2 correspondings to job J_1 and it has four alternatives $P_i\ (i = 1,2,3,4)$ and five attributes $w_j(j = 1,2, \dots 5)$. Assume that the characteristic information of the alternatives $P_i\ (i = 1,2,3,4)$ are represented by the IVIFNs, as shown in the interval-valued intuitionistic fuzzy decision matrix $\tilde{R}' = (\tilde{r}'_{ij})_{4\times5}$

Table 5: Interval-valued intuitionistic fuzzy decision matrix $\tilde{R}'$

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	w_1	w_2	J_1	w_3	w_4
P_1	([0.6,0.3].[0.2,0.4])	([0.7,0.6],[0.3,0.1])	([0.2,0.1],[0.7,0.8])	([0.2,0.6],[0.3,0.4])	([0.9,0.7],[0.1,0.1])
P_2	([0.4,0.5],[0.3,0.4])	([0.6,0.7],[0.1,0.2])	([0.2,0.3],[0.3,0.7])	([0.3,0.4],[0.1,0.2])	([0.4,0.6],[0.4,0.3])
P_3	([0.3,0.6],[0.3,0.4])	([0.6,0.7],[0.3,0.2])	([0.1,0.3],[0.4,0.5])	([0.7,0.6],[0.2,0.3])	([0.6,0.5],[0.3,0.4])
P_4	([0.7,0.4],[0.3,0.2])	([0.7,0.9],[0.1,0.1])	([0.0,0.4],[0.3,0.4])	([0.7,0.8],[0.2,0.1])	([0.6,0.5],[0.4,0.3])

J_1: ([max (0.2, 0.2, 0.1, 0.0), (0.1, 0.3, 0.3, 0.4)], [min (0.7, 0.3, 0.4, 0.3), (0.8, 0.7, 0.5, 0.4)])

J_1: ([0.2, 0.4], [0.3, 0.4])

Following similar process of team Y_1 we get the ideal priorities of the persons of team Y_2 corresponding to job J_1 represented by $P_4 > P_1 > P_3 > P_2$.

b) Consider the cost values of the persons of the team Y_2 to the job J_2 as P_1: ([0.0, 0.1], [0.5,0.6]), P_2: ([0.4, 0.0], [0.3,0.4]), P_3: ([0.1, 0.2], [0.8,0.7]) and P_4: ([0.3, 0.1], [0.7,0.5]) and the remaining values of attributes are as in Table 5. The ideal priorities of the persons of team Y_2 corresponding to job J_2 represented by $P_1 > P_4 > P_3 > P_2$.

c) Consider the cost values of the persons of the team Y_2 to the job J_3 as P_1: ([0.2, 0.1], [0.1,0.2]), P_2: ([0.2, 0.0], [0.8,0.2]), P_3: ([0.1, 0.2], [0.2,0.7]) and P_4: ([0.5, 0.4], [0.4,0.5]) and the remaining values of attributes are as in Table 5. The ideal priorities of the persons of team Y_2 corresponding to job J_3 represented by $P_4 > P_2 > P_3 > P_1$

d) Consider the cost values of the persons of the team Y_2 to the job J_4 as P_1: ([0.4, 0.3], [0.6,0.2]), P_2: ([0.3, 0.2], [0.4,0.7]), P_3: ([0.6, 0.3], [0.5,0.4]) and P_4: ([0.4, 0.3], [0.6,0.4]) and the remaining values of attributes are as in Table 5. The ideal priorities of the persons of team Y_2 corresponding to job J_4 represented by $P_4 > P_1 > P_3 > P_2$.

e) Consider the cost values of the persons of the team Y_2 to the job J_5 as $P_1:([0.3, 0.2], [0.7,0.2])$, $P_2:([0.1, 0.2], [0.4,0.3])$, $P_3:([0.1, 0.1], [0.5,0.4])$ and $P_4:([0.0, 0.4], [0.4,0.5])$ and the remaining values of attributes are as in Table 5. The ideal priorities of the persons of team Y_2 corresponding to job J_5 represented by $P_4 > P_1 > P_3 > P_2$.

Team Y_3

a) Let us consider team Y_3 correspondings to job J_1 and it has four alternatives $P_i\,(i = 1,2,3,4)$ and five attributes $w_j\,(j = 1,2, \dots 5)$. Assume that the characteristic information of the alternatives $P_i\ (i = 1,2,3,4)$ are represented by the IVIFNs, as shown in the interval-valued intuitionistic fuzzy decision matrix $\widetilde{R}' = \left(\tilde{r}'_{ij}\right)_{4\times 5}$

Table 6: Interval-valued intuitionistic fuzzy decision matrix $\widetilde{R}'$

	w_1	w_2	J_1	w_3	w_4
P_1	([0.4,0.3].[0.4,0.1])	([0.7,0.4],[0.4,0.3])	([0.3,0.1],[0.5,0.4])	([0.5,0.6],[0.3,0.2])	([0.8,0.5],[0.2,0.1])
P_2	([0.7,0.6],[0.3,0.2])	([0.3,0.4],[0.5,0.1])	([0.3,0.2],[0.7,0.4])	([0.1,0.5],[0.1,0.4])	([0.4,0.6],[0.4,0.3])
P_3	([0.7,0.1],[0.3,0.2])	([0.9,0.7],[0.1,0.1])	([0.3,0.2],[0.1,0.6])	([0.8,0.2],[0.2,0.1])	([0.6,0.5],[0.3,0.4])
P_4	([0.2,0.3],[0.3,0.1])	([0.1,0.6],[0.3,0.2])	([0.5,0.4],[0.4,0.5])	([0.6,0.2],[0.3,0.1])	([0.5,0.5],[0.4,0.3])

J_1: ([max (0.3, 0.3, 0.3, 0.5), (0.1, 0.2, 0.2, 0.4)], [min (0.5, 0.7, 0.1, 0.4), (0.4, 0.4, 0.6, 0.5)])

$J_1:([0.5, 0.4], [0.1, 0.4])$

Following similar process of team Y_1 we get the ideal priorities of the persons of team Y_3 corresponding to job J_1 represented by $P_3 > P_1 > P_2 > P_4$.

b) Consider the cost values of the persons of the team Y_3 to the job J_2 as $P_1:([0.1, 0.3], [0.3,0.2])$, $P_2:([0.1, 0.5], [0.4,0.5])$, $P_3:([0.4, 0.3], [0.3,0.6])$ and $P_4:([0.3, 0.4], [0.6,0.2])$ and

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the remaining values of attributes are as in Table 6. The ideal priorities of the persons of team Y_3 corresponding to job J_2 represented by $P_3 > P_1 > P_2 > P_4$.

c) Consider the cost values of the persons of the team Y_3 to the job J_3 as $P_1: ([0.1, 0.1], [0.1,0.2])$, $P_2: ([0.1, 0.2], [0.3,0.2])$, $P_3: ([0.4, 0.5], [0.5,0.4])$ and $P_4: ([0.3, 0.4], [0.6,0.2])$ and the remaining values of attributes are as in Table 6. The ideal priorities of the persons of team Y_3 corresponding to job J_3 represented by $P_3 > P_1 > P_2 > P_4$.

d) Consider the cost values of the persons of the team Y_3 to the job J_4 as $P_1: ([0.5, 0.4], [0.4,0.1])$, $P_2: ([0.0, 0.3], [0.6,0.3])$, $P_3: ([0.3, 0.0], [0.4,0.5])$ and $P_4: ([0.4, 0.3], [0.6,0.4])$ and the remaining values of attributes are as in Table 6. The ideal priorities of the persons of team Y_3 corresponding to job J_4 represented by $P_3 > P_1 > P_2 > P_4$.

e) Consider the cost values of the persons of the team Y_3 to the job J_5 as $P_1: ([0.5, 0.1], [0.2,0.5])$, $P_2: ([0.3, 0.2], [0.6, 0.7])$, $P_3: ([0.0, 0.4], [0.6,0.2])$ and $P_4: ([0.5, 0.4], [0.4,0.5])$ and the remaining values of attributes are as in Table 6. The ideal priorities of the persons of team Y_3 corresponding to job J_5 represented by $P_3 > P_1 > P_2 > P_4$.

Team Y_4

a) Let us consider team Y_4 correspondings to job J_1 and it has four alternatives $P_i (i = 1,2,3,4)$ and five attributes $w_j (j = 1,2, \dots 5)$. Assume that the characteristic information of the alternatives P_i $(i = 1,2,3,4)$ are represented by the IVIFNs, as shown in the interval-valued intuitionistic fuzzy decision matrix $\widetilde{R}' = (\tilde{r}'_{ij})_{4\times5}$

Table 7: Interval-valued intuitionistic fuzzy decision matrix $\widetilde{R}'$

	w_1	w_2	J_1	w_3	w_4
P_1	([0.3,0.4].[0.3,0.3])	([0.5,0.6],[0.1,0.2])	([0.1.,0.2],[0.3,0.2])	([0.6,0.7],[0.2,0.3])	([0.7,0.6],[0.2,0.3])

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P_2	([0.5,0.6],[0.1,0.2])	([0.2,0.3],[0.2,0.1])	([0.1,0.0],[0.2,0.1])	([0.5,0.8],[0.2,0.1])	([0.8,0.7],[0.1,0.2])
P_3	([0.7,0.3],[0.3,0.2])	([0.5,0.7],[0.3,0.3])	([0.3,0.2],[0.4,0.3])	([0.6,0.5],[0.4,0.3])	([0.6,0.2],[0.2,0.4])
P_4	([0.7,0.3],[0.3,0.1])	([0.5,0.6],[0.3,0.4])	([0.2,0.3],[0.6,0.4])	([0.8,0.6],[0.2,0.1])	([0.1,0.4],[0.3,0.1])

J_1: ([max (0.1, 0.1, 0.3, 0.2), (0.2, 0.0, 0.2, 0.3)], [min (0.3, 0.2, 0.4, 0.6), (0.2, 0.1, 0.3, 0.4)])

$J_1: ([0.3, 0.3], [0.2, 0.1])$

Following similar process of team Y_1 we get the ideal priorities of the persons of team Y_4 corresponding to job J_1 represented by $P_2 > P_1 > P_4 > P_3$.

b) Consider the cost values of the persons of the team Y_4 to the job J_2 as $P_1: ([0.1, 0.2], [0.2, 0.3])$, $P_2: ([0.3, 0.2], [0.4, 0.2])$, $P_3: ([0.3, 0.2], [0.6, 0.2])$ and $P_4: ([0.2, 0.4], [0.5, 0.4])$ and the remaining values of attributes are as in Table 7. The ideal priorities of the persons of team Y_4 corresponding to job J_2 represented by $P_2 > P_1 > P_4 > P_3$.

c) Consider the cost values of the persons of the team Y_4 to the job J_3 as $P_1: ([0.2, 0.1], [0.1, 0.2])$, $P_2: ([0.1, 0.1], [0.9, 0.6])$, $P_3: ([0.2, 0.0], [0.1, 0.2])$ and $P_4: ([0.2, 0.3], [0.6, 0.4])$ and the remaining values of attributes are as in Table 7. The ideal priorities of the persons of team Y_4 corresponding to job J_3 represented by $P_2 > P_1 > P_3 > P_4$.

d) Consider the cost values of the persons of the team Y_4 to the job J_4 as $P_1: ([0.1, 0.2], [0.5, 0.2])$, $P_2: ([0.1, 0.2], [0.7, 0.2])$, $P_3: ([0.3, 0.2], [0.5, 0.4])$ and $P_4: ([0.3, 0.3], [0.5, 0.4])$ and the remaining values of attributes are as in Table 7. The ideal priorities of the persons of team Y_4 corresponding to job J_4 represented by $P_2 > P_1 > P_4 > P_3$.

e) Consider the cost values of the persons of the team Y_4 to the job J_5 as $P_1: ([0.3, 0.2], [0.5, 0.4])$, $P_2: ([0.5, 0.4], [0.4, 0.5])$, $P_3: ([0.3, 0.2], [0.5, 0.2])$ and $P_4: ([0.3, 0.4], [0.6, 0.4])$ and

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the remaining values of attributes are as in Table 7. The ideal priorities of the persons of team Y_4 corresponding to job J_5 represented by $P_2 > P_1 > P_4 > P_3$.

Team Y_5

a) Let us consider team Y_5 correspondings to job J_1 and it has four alternatives $P_i\,(i = 1,2,3,4)$ and five attributes $w_j\,(j = 1,2,\dots 5)$. Assume that the characteristic information of the alternatives $P_i\,(i = 1,2,3,4)$ are represented by the IVIFNs, as shown in the interval-valued intuitionistic fuzzy decision matrix $\tilde{R}' = (\tilde{r}'_{ij})_{4\times5}$

Table 8: Interval-valued intuitionistic fuzzy decision matrix $\tilde{R}'$

	w_1	w_2	J_1	w_3	w_4
P_1	([0.6,0.4].[0.3,0.4])	([0.4,0.6],[0.3,0.2])	([0.2,0.1],[0.7,0.8])	([0.5,0.6],[0.4,0.3])	([0.5,0.7],[0.3,0.3])
P_2	([0.5,0.6],[0.2.,0.2])	([0.5,0.7],[0.2,0.3])	([0.4,0.2],[0.3,0.4])	([0.3,0.4],[0.2,0.3])	([0.9,0.6],[0.1,0.1])
P_3	([0.6,0.5],[0.3,0.3])	([0.5,0.6],[0.3,0.4])	([0.3,0.2],[0.7,0.4])	([0.1,0.5],[0.2,0.3])	([0.7,0.2],[0.2,0.3])
P_4	([0.5,0.3],[0.3,0.2])	([0.5,0.6],[0.3,0.1])	([0.3,0.4],[0.3,0.5])	([0.7,0.6],[0.2,0.2])	([0.1,0.5],[0.3,0.1])

J_1: ([max (0.2, 0.4, 0.3, 0.3), (0.1, 0.2, 0.2, 0.4)], [min (0.7, 0.3, 0.7, 0.3), (0.8, 0.4, 0.4, 0.5)])

J_1: ([0.4, 0.4], [0.3, 0.4]).

Following similar process of team Y_1 we get the ideal priorities of the persons of team Y_5 corresponding to job J_1 represented by $P_2 > P_1 > P_4 > P_3$.

b) Consider the cost values of the persons of the team Y_5 to the job J_2 as P_1: ([0.2, 0.1], [0.6,0.8]), P_2: ([0.3, 0.2], [0.4,0.2]), P_3: ([0.2, 0.1], [0.8,0.6]) and P_4: ([0.3, 0.4], [0.5,0.5]) and the remaining values of attributes are as in Table 8. The ideal priorities of the persons of team Y_5 corresponding to job J_2 represented by $P_2 > P_3 > P_1 > P_4$.

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c) Consider the cost values of the persons of the team Y_5 to the job J_3 as $P_1:([0.2,0.1],[0.7,0.8])$, $P_2:([0.5,0.2],[0.1,0.0])$, $P_3:([0.3,0.2],[0.3,0.4])$ and $P_4:([0.3,0.4],[0.3,0.4])$ and the remaining values of attributes are as in Table 8. The ideal priorities of the persons of team Y_5 corresponding to job J_3 represented by $P_2 > P_1 > P_4 > P_3$.

d) Consider the cost values of the persons of the team Y_5 to the job J_4 as $P_1:([0.2,0.1],[0.7,0.6])$, $P_2:([0.4,0.2],[0.3,0.5])$, $P_3:([0.3,0.2],[0.7,0.3])$ and $P_4:([0.3,0.4],[0.3,0.6])$ and the remaining values of attributes are as in Table 8. The ideal priorities of the persons of team Y_5 corresponding to job J_4 represented by $P_2 > P_1 > P_3 > P_4$.

e) Consider the cost values of the persons of the team Y_5 to the job J_5 as $P_1:([0.2,0.1],[0.5,0.7])$, $P_2:([0.2,0.4],[0.4,0.2])$, $P_3:([0.3,0.2],[0.4,0.4])$ and $P_4:([0.3,0.4],[0.4,0.5])$ and the remaining values of attributes are as in Table 8. The ideal priorities of the persons of team Y_5 corresponding to job J_5 represented by $P_2 > P_1 > P_4 > P_3$.

Team pool Y_6

a) Let us consider team Y_6 correspondings to job J_1 and it has four alternatives $P_i\,(i = 1,2,3,4)$ and five attributes $w_j\,(j = 1,2,\dots 5)$. Assume that the characteristic information of the alternatives $P_i\;(i = 1,2,3,4)$ are represented by the IVIFNs, as shown in the interval-valued intuitionistic fuzzy decision matrix $\tilde{R}' = \left(\tilde{r}'_{ij}\right)_{4\times5}$.

Table 9: Interval-valued intuitionistic fuzzy decision matrix $\tilde{R}'$

	w_1	w_2	J_1	w_3	w_4
P_1	([0.3,0.4].[0.4,0.2])	([0.8,0.9],[0.1,0.1])	([0.1,0.2],[0.2,0.4])	([0.4,0.5],[0.3,0.5])	([0.2,0.4],[0.3,0.1])
P_2	([0.5,0.7],[0.1,0.3])	([0.4,0.7],[0.2,0.3])	([0.2,0.2],[0.3,0.5])	([0.6,0.8],[0.1,0.2])	([0.3,0.4],[0.2,0.3])
P_3	([0.2,0.4],[0.1,0.2])	([0.4,0.5],[0.2,0.4])	([0.1,0.2],[0.7,0.8])	([0.4,0.6],[0.2,0.3])	([0.6,0.3],[0.2,0.4])

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P_4	([0.7,0.8],[0.1,0.2])	([0.5,0.7],[0.1,0.2])	([0.4,0.5],[0.5,0.4])	([0.4,0.5],[0.1,0.2])	([0.3,0.2],[0.0,0.2])

Following similar process of team Y_1 we get the ideal priorities of the persons of team Y_6 corresponding to job J_1 represented by $P_1 > P_2 > P_4 > P_3$.

b) Consider the cost values of the persons of the team Y_6 to the job J_2 as

$P_1:([0.1,0.2],[0.1,0.3])$, $P_2:([0.1,0.2],[0.4,0.5])$, $P_3:([0.1,0.4],[0.5,0.6])$ and $P_4:([0.4,0.3],[0.6,0.4])$ and the remaining values of attributes are as in Table 9. The ideal priorities of the persons of team Y_6 corresponding to job J_2 represented by $P_1 > P_4 > P_2 > P_3$.

c) Consider the cost values of the persons of the team Y_6 to the job J_3 as

$P_1:([0.2,0.2],[0.3,0.4])$, $P_2:([0.2,0.3],[0.5,0.5])$, $P_3:([0.1,0.2],[0.3,0.4])$ and $P_4:([0.4,0.2],[0.3,0.6])$ and the remaining values of attributes are as in Table 9. The ideal priorities of the persons of team Y_6 corresponding to job J_3 represented by $P_1 > P_4 > P_2 > P_3$.

d) Consider the cost values of the persons of the team Y_6 to the job J_4 as

$P_1:([0.1,0.2],[0.4,0.3])$, $P_2:([0.2,0.1],[0.5,0.3])$, $P_3:([0.1,0.2],[0.6,0.7])$ and $P_4:([0.1,0.2],[0.3,0.2])$ and the remaining values of attributes are as in Table 9. The ideal priorities of the persons of team Y_6 corresponding to job J_4 represented by $P_4 > P_2 > P_1 > P_3$.

e) Consider the cost values of the persons of the team Y_6 to the job J_5 as

$P_1:([0.1,0.2],[0.4,0.3])$, $P_2:([0.3,0.2],[0.6,0.4])$, $P_3:([0.1,0.2],[0.6,0.8])$ and $P_4:([0.2,0.3],[0.3,0.4])$ and the remaining values of attributes are as in Table 9. The ideal priorities of the persons of team Y_6 corresponding to job J_5 represented by $P_4 > P_3 > P_2 > P_1$.

Final Table 10 R_1': Cost matrix of the teams corresponding to jobs

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	J_1	J_2	J_3	J_4	J_5
Y_1	([0.2,0.3].[0.4,0.5])	([0.4,0.3],[0.1,0.5])	([0.4,0.5],[0.2,0.4])	([0.4,0.2],[0.3,0.2])	([0.5,0.4],[0.2,0.3])
Y_2	([0.2,0.4],[0.3,0.4])	([0.4,0.2],[0.3,0.4])	([0.5,0.4],[0.1,0.2])	([0.6,0.3],[0.4,0.2])	([0.3,0.4],[0.4,0.2])
Y_3	([0.5,0.4],[0.1,0.4])	([0.4,0.5],[0.3,0.2])	([0.4,0.5],[0.1,0.2])	([0.5,0.4],[0.4,0.1])	([0.5,0.4],[0.2,0.2])
Y_4	([0.3,0.3],[0.2,0.1])	([0.3,0.4],[0.2,0.2])	([0.2,0.3],[0.1,0.2])	([0.3,0.3],[0.5,0.2])	([0.5,0.4],[0.4,0.2])
Y_5	([0.4,0.4],[0.3,0.4])	([0.3,0.4],[0.4,0.2])	([0.5,0.4],[0.1,0.0])	([0.4,0.4],[0.3,0.3])	([0.3,0.4],[0.4,0.2])

Suppose the person P_2 is absent in the team Y_4. Choose the person from the pool whose priority is higher corresponding to job J_1 and replace the value of above said person in the team Y_4 with the place P_2 corresponding to job J_1. Find the maximum of acceptance and minimum of rejection of the cost to do job J_1 by the person of the team Y_2. Put the value in the table $R_1{}'$ with respect to the team Y_4 corresponding to the job J_1. Similarly we do for other values of J_2, J_3, J_4 *and* J_5 by the team Y_4. Suppose more than one person are absent replacement is made from the hierarchical order.

Table 11 : After break down the cost matrix $R_1{}'$

	J_1	J_2	J_3	J_4	J_5
Y_1	([0.2,0.3].[0.4,0.5])	([0.4,0.3],[0.1,0.5])	([0.4,0.5],[0.2,0.4])	([0.4,0.2],[0.3,0.2])	([0.5,0.4],[0.2,0.3])
Y_2	([0.2,0.4],[0.3,0.4])	([0.4,0.2],[0.3,0.4])	([0.5,0.4],[0.1,0.2])	([0.6,0.3],[0.4,0.2])	([0.3,0.4],[0.4,0.2])
Y_3	([0.5,0.4],[0.1,0.4])	([0.4,0.5],[0.3,0.2])	([0.4,0.5],[0.1,0.2])	([0.5,0.4],[0.4,0.1])	([0.5,0.4],[0.2,0.2])
Y_4	([0.3,0.3],[0.2,0.1])	([0.3,0.4],[0.1,0.2])	([0.2,0.3],[0.1,0.2])	([0.3,0.3],[0.3,0.2])	([0.3,0.4],[0.3,0.2])
Y_5	([0.4,0.4],[0.3,0.4])	([0.3,0.4],[0.4,0.2])	([0.5,0.4],[0.1,0.0])	([0.4,0.4],[0.3,0.3])	([0.3,0.4],[0.4,0.2])

Utilizing equation (1) calculate the score matrix $R_1{}'$

Table 12: Score matrix of $R_1{}'$

	J_1	J_2	J_3	J_4	J_5
Y_1	-0.2	0.05	0.15	0.05	0.2
Y_2	-0.05	-0.05	0.3	0.15	0.05
Y_3	0.2	0.2	0.3	0.2	0.25
Y_4	0.15	0.2	0.1	-0.05	0.05
Y_5	0.05	0.05	0.4	0.1	0.05

The table 12 is the cost matrix of the assignment problem in the maximization form and it can be solved by Hungarian method or by using any standard software.

The optimal assignment without break down is

1st job is assigned to the 1st team.
2nd job is assigned to the 2nd team.
3rd job is assigned to the 3rd team.
4th job is assigned to the 4th team.
5th job is assigned to the 5th team.

The optimal assignment after break down is

1st job is assigned to the 1st team.
2nd job is assigned to the 2nd team.
3rd job is assigned to the 4th team.
4th job is assigned to the 3rd team.
5th job is assigned to the 5th team.

VII. CONCLUSION

In this paper a new real life interval-valued intuitionistic fuzzy assignment model with replacement is proposed. Two stages of solution procedure are discussed with intuitionistic fuzzy aggregation operators. By an example ideal priorities of the professionals of teams are found and replacement is made where it is necessary by means of the priorities to do the job in time.

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