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Physical Interpretation of DCOPF based Lagrange Multipliers- LMP, Shadow Prices.

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Abstract-*This paper examines with a few examples, the Lagrange multipliers- Locational Marginal Price (LMP), Shadow Prices which are scalar quantities for unconstrained and constrained DC power flow differentiable functions. They are evaluated through optimization process by increasing the constrained parameter a unit. Lagrange multipliers deal with both equality and inequality constraints which restrict the feasible region to points by lying on some surface inside the real part. Lagrangian function is applied to a linear programming problem (LPP) which is defined as the problem of maximizing or minimizing a linear function subject to linear constraints. A few case studies are provided to calculate Lagrange multipliers for a 3-bus, 5-bus and 8-bus unconstrained and constrained power flow in electrical power systems which are popularly called as 'shadow prices' when line congestion takes place in transmitting power. In this paper it reveals that the calculated LMP at any bus is the Lagrange multiplier for the bus equality constraint in OPF.*

Index Terms: Constrained Optimization, DCOPF, Lagrange Multipliers, Linear Programming (LP), Locational Marginal Prices (LMP), Shadow Prices.

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1 Introduction

Joseph-Louis Lagrange devised a beautiful method that preserves the symmetry of the variables without elimination. The Lagrangian function is formed as $L(x) = f(x) + \Sigma_j^l \lambda_j h_j(x)$ where λ_j is a scalar multiplier associated with constraint h_j. If x^* is a stationary point (minimum or maximum or a point of inflection), then it is necessary that the following conditions be satisfied can be written as

$$\nabla_x L = 0 \text{ and}$$
$$\mathbf{h}=0$$

Optimality conditions [1] can be used to obtain canditate solutions or to verify if a given point x^* is a minimum point. Lagrangian multiplier [2] is the rate at which we could increase the Lagrangian if we were to raise the constraint from zero.

The concept of Locational Marginal Price known as nodal price was first developed by Schweppe [3]. LMPs can be derived either by ACOPF model or by DCOPF model as cited by Momoh. The linearized ACOPF model is DCOPF model & is widely used for LMP calculation in power market operation [4] as cited by Litvinov in 2004. There are two forms of DCOPF models-'full structured' & 'reduced form' cited by Shahidepour & Li [5]. The reduced form DCOPF model solves out for voltage angles using real power balance equations. Though ACOPF model is more accurate than the DCOPF model, it is 60 times slower than the DC model as cited

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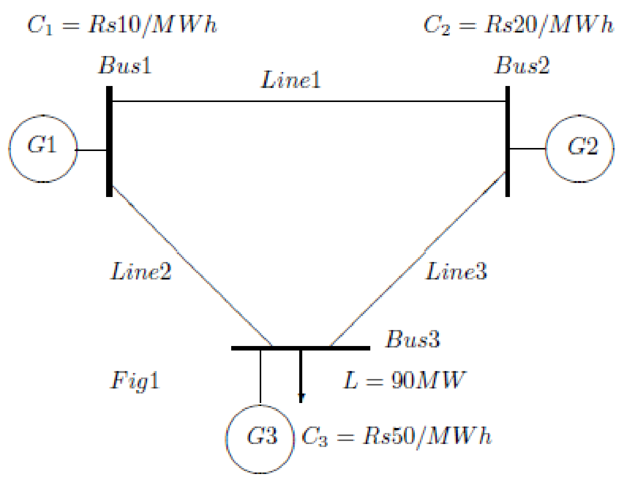
by Overbye. Locational Marginal Price (LMP) [6] is defined as the marginal cost of supplying the next increment of electric energy at a specific bus while considering the generation marginal cost and the physical aspects of the transmission system. LMP-based market [7] pricing approach is to manage the efficient use of transmission system when congestion [8] occurs on bulk power grids. With the restructuring, into the electric power industry, the price of electricity has become the focus of all activities in the power markret and LMPs provide important economic signals [9] that fully reflect both system and market operations at a specified time. In other words LMPs are the 'shadow prices' of the real power balance equality constraints of an optimization problem [10] that maximizes the total social welfare function, based on the offer and bid functions [11] of the sellers and buyers respectively, for a specified point of time.
LMP is popularly defined as the change in production cost to optimally deliver an increment of load at the locations while satisfying all the constraints. Neglecting losses,

$$LMP = LMP_{energy} + LMP_{congestion}$$

2 Evaluating the LMPs and the shadow prices through LP problems applied in Power system.

2.1 Example 1

A case Study-I, with 3 buses and 3 lines.



$Fig1$

Given data-
In a 3 bus system, Fig1, the conditions are,
1. Load=90MW
2. Gen Max.Capacity=120MW
3. Line Max Capacity=50MW
4. All lines have equal reactances
5. **Bus no 3 is the reference bus.**

$$\text{So, } Y_{BUS}=\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

$$\text{\& X=Sensitivity Matrix}=\begin{bmatrix} 2/3 & 1/3 & 0 \\ 1/3 & 2/3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

is calculated by appending with zeros at the 3rd row & 3rd column as bus no 3 is the reference bus and inverting the rest of the matrix of the Y_{BUS}.

Applying DCOPF analysis,

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 2/3 & 1/3 & 0 \\ 1/3 & 2/3 & 0 \\ 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} P_{g1} \\ P_{g2} \\ P_{g3} - 90 \end{bmatrix}$$

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So,

$$P_{13}=2/3*P_{g1}+1/3*P_{g2}$$
$$P_{23}=1/3*P_{g1}+2/3*P_{g2}$$
$$P_{12}=1/3*P_{g1}-1/3*P_{g2}$$

where P_{13}, P_{23}, P_{12}, P_{g1}, P_{g2}, P_{g3}, θ_1, θ_2, θ_3 are the line flows, generations & bus voltage angles.

For unconstrained case and no line limits

Let $P_{g1}=x_1$, $P_{g2}=x_2$, $P_{g3}=x_3$

$$\text{Minimize } 10*x_1+20*x_2+50*x_3$$
$$\text{s.t } x_1+x_2+x_3=90$$
$$x_1 \geq 0$$
$$x_1 \leq 120$$
$$x_2 \geq 0$$
$$x_2 \leq 120$$
$$x_3 \geq 0$$
$$x_3 \leq 120$$

Results are x_1=90, x_2=0, x_3=0. So, P_{12}=30, P_{13}=60, P_{23}=30.

As the line limit is imposed, the constrained line P_{13} is set to be at 50MW.

Minimize $10*x_1+20*x_2+50*x_3$

s.t $x_1+x_2+x_3=90$
$2/3*x_1+1/3*x_2 \leq 50$
$x_1 \geq 0$
$x_1 \leq 120$
$x_2 \geq 0$
$x_2 \leq 120$
$x_3 \geq 0$
$x_3 \leq 120$

Results are x_1=60, x_2=30, x_3=0. So, P_{12}=10, P_{13}=50, P_{23}=40.

Through Lagrange's method the same result is obtained.

L=$10*x_1+20*x_2+50*x_3$
$+\lambda*(-x_1-x_2-x_3+90)$
$+\mu*(2/3*x_1+1/3*x_2-50)$

Setting, $\frac{dL}{dx_1}=\frac{dL}{dx_2}=\frac{dL}{dx_3}=\frac{dL}{d\lambda}=\frac{dL}{d\mu}=0$

where λ, μ are the Lagrange multipliers, On solving,

$10-\lambda+2/3*\mu=0$
$20-\lambda+1/3*\mu=0$
$x_1+x_2+x_3=90$
$2/3*x_1+1/3*x_2=50$

Results are x_1=60, x_2=30, x_3=0 is the only feasible solution, other values x_1=0, x_2=150, x_1=75, x_2=0 are discarded due to limitations. λ=30, μ=30.

Increasing the load from 90MW to 91MW, the change in cost is 30. So λ=30. Increasing the line limit to 51MW, the change in cost is 30. So μ=30.

To calculate the 'shadow price', by increasing the constraint line to 1MW the change in cost is calculated.

$$\text{Minimize } 10*x_1+20*x_2+50*x_3$$
$$\text{s.t } x_1+x_2+x_3=90$$
$$2/3*x_1+1/3*x_2 \leq 50+1$$
$$x_1 \geq 0$$
$$x_1 \leq 120$$
$$x_2 \geq 0$$
$$x_2 \leq 120$$
$$x_3 \geq 0$$
$$x_3 \leq 120$$

Results are x_1=63, x_2=27, x_3=0. So, decrease in cost=(63-60)*10+(27-30)*20=-30. So μ=30.

To find the LMPs by an extra load at buses,

$$\text{Minimize } 10*x_1+20*x_2+50*x_3$$
$$\text{s.t } x_1+x_2+x_3=90+1$$
$$2/3*x_1+1/3*(x_2-1) \leq 50$$
$$x_1 \geq 0$$
$$x_1 \leq 120$$
$$x_2 \geq 0$$
$$x_2 \leq 120$$
$$x_3 \geq 0$$
$$x_3 \leq 120$$

Results are x_1=60, x_2=31, x_3=0.
So,LMP_2=Change in cost=20.

$$\text{Minimize } 10*x_1+20*x_2+50*x_3$$
$$\text{s.t } x_1+x_2+x_3=90+1$$
$$2/3*x_1+1/3*x_2 \leq 50$$
$$x_1 \geq 0$$
$$x_1 \leq 120$$
$$x_2 \geq 0$$
$$x_2 \leq 120$$
$$x_3 \geq 0$$
$$x_3 \leq 120$$

Results are x_1=59, x_2=32, x_3=0.
So,LMP_3=Change in cost=30.
Similarly LMP_1=10.

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2.2 Example 2

A case Study-II, with 5 buses and 6 lines.

A PJM-5 bus study, Fig2, six lines test system is a standard test case. The system is roughly divided into two areas, a generation center consisting of bus 1 and 5 with three low cost generation units and a load center consisting of buses 2, 3 and 4 and two high cost generating units.

Table-I

Line Data	1-2	1-4	1-5	2-3	3-4	5-4
X	2.81	3.04	0.64	1.08	2.97	2.97
Limit	999	999	999	999	999	240

Table-II

Gen	P_{min}	P_{max}	**Marginal Cost**
G_1	0	110	14
G_2	0	100	15
G_3	0	520	30
G_4	0	200	35
G_5	0	600	10

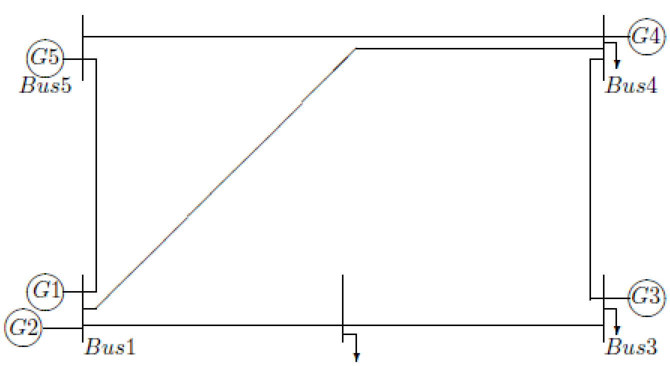

Fig2

Loads are inelastic and are 300MW each. **Bus 4 is taken as the slack bus.**

$$X=\begin{bmatrix}1.3303 & 0.7854 & 0.5760 & 0 & 1.0945\\ 0.7854 & 2.1226 & 1.5566 & 0 & 0.6462\\ 0.5760 & 1.5566 & 1.9335 & 0 & 0.4739\\ 0 & 0 & 0 & 0 & 0\\ 1.0945 & 0.6462 & 0.4739 & 0 & 1.4270\end{bmatrix}$$ where X

is the Sensitivity matrix.
Let P_{15}, P_{14}, P_{12}, P_{23}, P_{34}, P_{54} are the line flows as P_{ij} represent the line flows from bus i to j.

Let $P_{g1}=x_1$, $P_{g2}=x_2$, $P_{g3}=x_3$, $P_{g4}=x_4$, $P_{g5}=x_5$

Minimize
$$14*x_1+15*x_2+30*x_3+35*x_4+10*x_5$$
s.t $x_1+x_2+x_3+x_4+x_5=900$, $x_1\geq 0$, $x_1\leq 110$, $x_2\geq 0$, $x_2\leq 100$, $x_3\geq 0$, $x_3\leq 520$, $x_4\geq 0$, $x_4\leq 200$, $x_5\geq 0$, $x_5\leq 600$, $P_{15}\leq 999$, $P_{14}\leq 999$, $P_{12}\leq 999$, $P_{23}\leq 999$, $P_{34}\leq 999$, $P_{54}\leq 240$
Results are x_1=110, x_2=100, x_3=0, x_4=116.0792 and x_5=573.9208. Total Cost=12,842.

To calculate the 'shadow price',

Minimize
$$14*x_1+15*x_2+30*x_3+35*x_4+10*x_5$$
s.t $x_1+x_2+x_3+x_4+x_5=900$, $x_1\geq 0$, $x_1\leq 110$, $x_2\geq 0$, $x_2\leq 100$, $x_3\geq 0$, $x_3\leq 520$, $x_4\geq 0$, $x_4\leq 200$, $x_5\geq 0$, $x_5\leq 600$, $P_{15}\leq 999$, $P_{14}\leq 999$, $P_{12}\leq 999$, $P_{23}\leq 999$, $P_{34}\leq 999$, $P_{54}\leq 240+1$

Results are x_1=110, x_2=100, x_3=0, x_4=113.9979 and x_5=576.0021.
So, 'shadow price' μ_{4-5}=(113.9979-116.0792)*35+(576.0021-573.99208)*10=-52.0325 i.e net savings or net decrease in cost.

To calculate the LMPs, for example LMP_1, put an extra fictious load at bus 1.

Minimize
$$14*x_1+15*x_2+30*x_3+35*x_4+10*x_5$$

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s.t $x_1 + x_2 + x_3 + x_4 + x_5 = 900 + 1$, $x_1 \geq 0$, $x_1 \leq 110$, $x_2 \geq 0$, $x_2 \leq 100$, $x_3 \geq 0$, $x_3 \leq 520$, $x_4 \geq 0$, $x_4 \leq 200$, $x_5 \geq 0$, $x_5 \leq 600$, $1/2.97 * (1.0945 * (x_1 + x_2 - 1) - 193.86 + 0.4739 * (x_3 - 300) + 1.4270 * x_5) \leq 240$

Results are x_1=110, x_2=100, x_3=0, x_4=116.3122 and x_5=574.6878

So, LMP_1= Change in cost=(116.3122-116.0792)*35+(574.6878-573.9208)*10=15.825 Similarly LMP_2=23.6775, LMP_3=26.6975, LMP_4=35, LMP_5=10.0

2.3 Example 3

A case Study-III, with 8 buses and 10 lines.

Generating Units Information

Table-III

Bus No.	c	b	a	P_{min}	P_{max}
1	0.0060	18	100	5	20
2	-	-	-	-	-
3	0.0041	17.02	145	5	70
4	0.0039	12	112	10	70
5	0.0040	10.1	110	10	90
6	0.0040	30	180	10	70
7	0.0035	9.3	100	20	200
8	-	-	-	-	-

where the units of c, b, a are Rs/MWh^2, Rs/MWh & Rs/h respectively.

Line Parameters Information

Table-IV

Line No.	From Bus	To Bus	X p.u	LineLimit
L1	1	2	0.03	200
L2	1	4	0.03	200
L3	2	3	0.011	200
L4	3	4	0.03	60
L5	4	5	0.008	22
L6	5	6	0.02	110
L7	6	1	0.02	75
L8	7	4	0.015	200
L9	7	8	0.022	200
L10	8	3	0.018	200

Bus 1 is the reference bus.

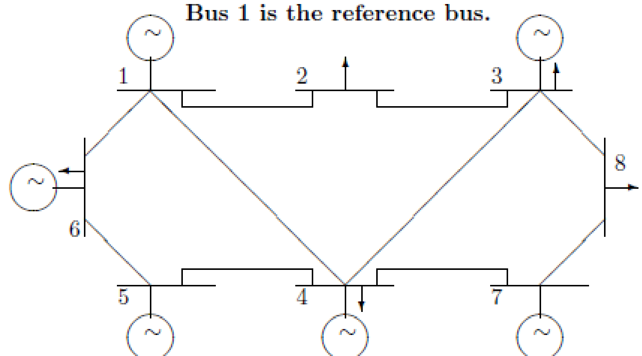

Fig3

$$X=10^{-3}*\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 18.6 & 14.4 & 7 & 5.9 & 2.9 & 9 & 12 \\ 0 & 14.4 & 19.7 & 9.6 & 8 & 4 & 12.3 & 16.4 \\ 0 & 7 & 9.6 & 14.1 & 11.8 & 5.9 & 12.9 & 11.1 \\ 0 & 5.9 & 8 & 11.8 & 16.5 & 8.2 & 10.8 & 9.2 \\ 0 & 2.9 & 4 & 5.9 & 8.2 & 14.1 & 5.4 & 4.6 \\ 0 & 9 & 12.3 & 12.9 & 10.8 & 5.4 & 23.7 & 17.4 \\ 0 & 12 & 16.4 & 11.1 & 9.2 & 4.6 & 17.4 & 26.8 \end{bmatrix}$$

Line flows of P_{45}, P_{56} gets overloaded.

LMP_1=6091.6-6068.8=22.8, LMP_2=6087.7-6068.8=18.9, LMP_3=6086.4-6068.8=17.6, LMP_4=6083.9-6068.8=15.1, LMP_5=6080.7-6068.8=11.9, LMP_6=6099.3-6068.8=30.5, LMP_7=6084.5-6068.8=15.7, LMP_8=6085.6-6068.8=16.8. Shadow Price= Decrease in cost=6042.6-6068.8=-26.2

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3 Conclusion

Locational Marginal Pricing (LMP) plays an important role in restructured power markets. Different DCOPF models are presented to prove that LMPs and the 'Shadow prices' are the Lagrange multipliers of the unconstrained and constrained power flow in the power system. The bus which has the highest LMP means the buyer who buys the power from this bus has to pay more and the bus which has the lowest LMP means every buyer wants to buy power from that bus and this crates congestion in that transmission lines. Therefore it is necessary to reduce the output of cheaper units and increase the output of more expensive units to reduce the congestion and to enhance the reliability and security of the transmission line.

Appendix

The loads at bus 2, 3, 4, 6 & 8 are 53.4, 71.2, 53.2, 186.9 & 84.55 MW respectively in Fig3. X is the sensitivity matrix.

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